01 Verdict

The geometry passes. The earlier presentation did not demonstrate its connection to the logo sufficiently.

The numbers were not fabricated. The detached square, gap and uniform black-bar thickness satisfy the stated plastic-number relations. Fresh checks parsed the delivered SVG itself; its largest checked dimension discrepancy is below 9×10^{-13} units for a frame side of 1000.

What was overstated: the first delivery showed a triangle spiral beside a formula-based logo without showing the transfers. That was not a demonstration that one connected spiral dictated the mark. The earlier record did identify the proportions as design choices; those choices are still choices. There is no established uniquely correct or aesthetically optimal logo.

Correction: retain the verified artwork, add an honestly registered connected spiral and show all four selected lengths transferred at scale 1. Each moved copy has a recorded rigid transformation. The construction record is rewritten so its conclusion matches that evidence.

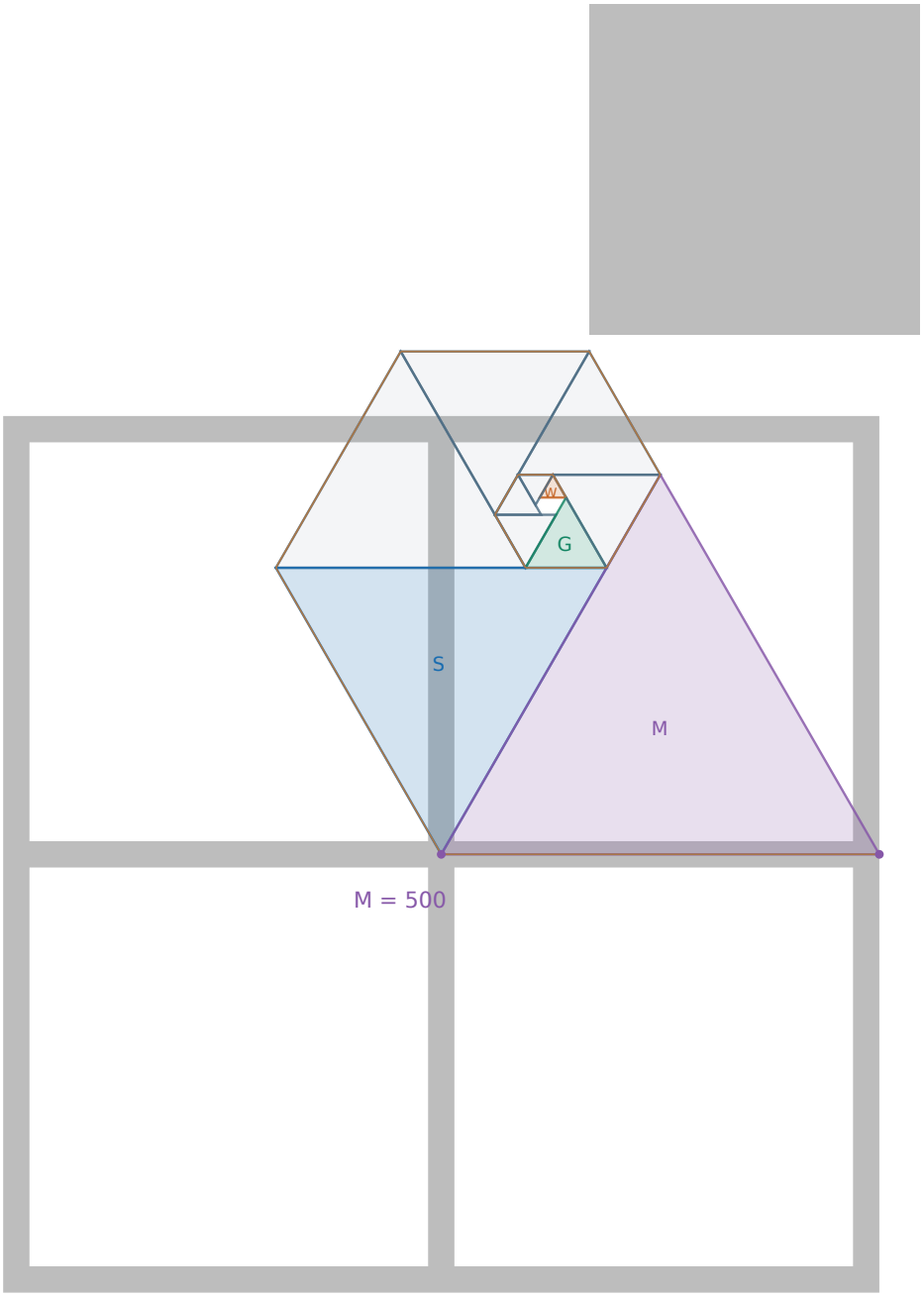
Claim	Result
Cubic and selected length identities	PASS, exact algebra
Equilateral triangles and shared edges	PASS, exact algebra
No interior overlap in source windows	PASS, exact separating-axis checks
Delivered SVG respects dimensions	PASS, within stated decimal tolerance
One spiral fixes every logo boundary	NOT ESTABLISHED; not claimed
Unique geometric or visual perfection	NOT ESTABLISHED; not claimed

Limits of this review. This is the same assistant reviewing its own work. The validator is separately written and does not import the original generator; it is not an independent human review. No new external expert endorsement is implied.

The SVG, vector PDF and clean PNG logo exports are retained. The revised material changes the evidence and explanation, not the accepted dimensions.

02 One connected spiral

The entire source uses one fixed placement. Triangle T0 spans half the frame’s outside width.

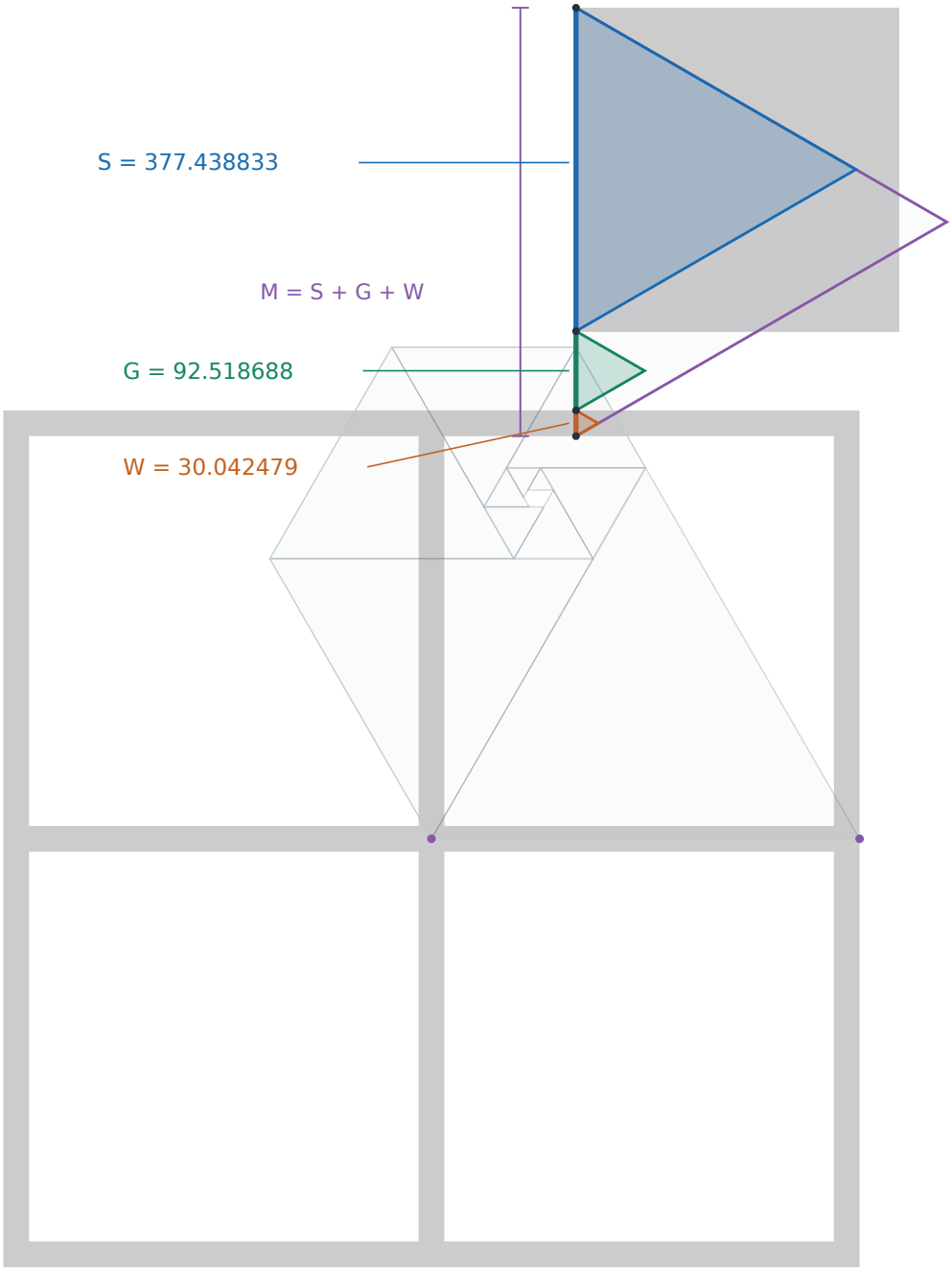


The triangle-side scale is $T_n = M\rho^n$. Its T0 edge is fixed from (500,500) to (1000,500), where D=1000 and M=D/2. Every other source triangle follows from that one placement. The overlay shows only n=-10 through 0; the largest side is 500 and the smallest is 30.042479. There is no unbounded outward growth.

This is the self-similar plastic limit of the Padovan construction. The floating square is deliberately visible: it is not bounded by this connected spiral. Its dimensions require the transfers shown on the next page.

03 Transparent length transfers

The copied triangles preserve their sizes. Their new positions are chosen and disclosed.



Blue: S is the floating-square side. **Green:** G is the clear gap. **Orange:** W is the black-bar thickness. **Purple:** M=S+G+W. The pale connected spiral remains in the source placement; the stronger triangles are moved copies.

All copies are rotated and translated with scale exactly 1 in the mathematical construction. Their target positions intentionally overlap. They no longer form one connected tiling there. No triangle is stretched or resized to disguise a mismatch.

04 Exact placement of the copies

Coordinates use x to the right and y downward. A denotes the floating square's upper-left corner.

```
A = (D+O-S, -G-S) = (668.820510689, -469.957520748)
```

Copy	Side	Target edge, from A	Rotation*
T0	M	A to A + (0,M)	90 deg
T-1	S	A to A + (0,S)	30 deg
T-6	G	A+(0,S) to A+(0,S+G)	90 deg
T-10	W	A+(0,S+G) to A+(0,M)	210 deg

*Rotations are measured from the canonical y-down source coordinates. Source translation is (500,500). Each copy translation is target_start minus the rotated canonical start. The complete six matrix coefficients for every copy are in review/overlay-transforms.json. Orthogonality, determinant +1 and the target endpoints are checked from the exported SVG.

```
rho^3 = rho + 1

rho^5 = rho^4 + 1

1 - 1/rho = rho^-5;   rho - 1 = rho^-4

S = M/rho

G = S - S/rho = M/rho^6

W = G(rho-1) = M/rho^10

S + G + W = S + rho G = rho S = M
```

The purple triangle side and the three contiguous colored edges end at the same lower point (A.x,W). The gap ends exactly at the outside top edge y=0; the orange edge ends at y=W, the inside bottom edge of the top border. These coincidences follow from the equations. They are not adjusted after rendering.

The transfer method constructs an orthogonal mark from equilateral-triangle lengths. Perpendiculars and bisections are separate, explicit square-grid operations. A diagram cannot prove that arbitrary aesthetic choices were forced by the Padovan recurrence.

05 The design choices

The constraints below define this particular reconstruction; the plastic number does not select them automatically.

Form. Retain a square frame, four equal square apertures and one detached square above the right side. Preserve horizontal and vertical edges and the source’s small positive overhang.

Scale. Choose outside frame side $D=1000$ arbitrary units. Use $M=D/2$ because the mark has a 2-by-2 grid. M is not the aperture side or the divider-to-divider pitch.

Selected proportions. Choose $S=M/\rho$, $G=S-S/\rho$ and $W=G(\rho-1)$. Their proximity to the source motivated the choices; no optimization or aesthetic theorem establishes them. They correspond to T-1, T-6 and T-10 in the limit scale.

Placement. Choose right overhang $O=G/2$. The square occupies $[D+O-S,D+O] \times [-G-S,-G]$. Halving G is a deliberate rule, not a property of Padovan triangles.

Four apertures. Use $C=(D-3W)/2$. Centre divider bands at $x=D/2$ and $y=D/2$. Every border and divider has width W. Remove four congruent C-by-C holes from the outer black square.

Appearance. Pure black, square corners, no text, no curves or effects. The master is one filled compound frame path plus the detached-square rectangle. The colored transparent construction overlays are separate explanatory files.

Canvas. Use clear space G around the complete ink bounds. Centre the ink bounding box, including the overhang, in that padding. No optical or ink-centroid shift is applied.

Spiral and copies. Choose the displayed source pose and the vertical target line through A for legibility and an auditable partition. Neither registration changes the logo. Copy rotations preserve lengths. The drawing stops at the smallest needed scale T-10.

06 Dimensions and reference

The original image already had four equal pixel apertures. The reconstruction was a controlled proportional change.

Quantity	Reference pixels	Rebuilt, same D=244
Outside frame side	244	244.000000
Detached-square side	95	92.095075
Vertical gap	24	22.574560
Right overhang	11	11.287280
Outside-border thickness	7	7.330365
Central-divider thickness	8	7.330365
Each clear square aperture	111	111.004453

Reference measurements use RGB channels below 128 and half-open pixel boxes. The frame is [145,389) × [159,403); the floating square is [305,400) × [40,135). A one-pixel difference between raster bar widths does not prove that the unknown original vector was geometrically wrong. The corrected claim concerns this delivered vector, not the unseen source design.

Master parameter	D=1000 units
S	377.438833123346
G	92.518687624320
W	30.042479252334
O	46.259343812160
C	454.936281121499

Let I0=(W,W+C) and I1=(2W+C,2W+2C). The frame is $[0,D]^2$ with each $I_i \times I_j$ removed. The SVG uses the even-odd fill rule. Its viewBox is (-G,-S-2G,D+O+2G,D+S+3G). These formulas specify every visible edge and its padding.

The mathematical definitions are exact. Exported SVG coordinates are finite decimals; the review checks a 3×10^{-12}-unit tolerance. PDF and PNG exports have their own finite rendering precision. Raster antialiasing is not exact Euclidean geometry and no minimum-size optical variant is claimed.

07 Triangle construction and proof

Integer Padovan triangles and the exact plastic limit are distinct constructions.

```
P0=P1=P2=1;    Pn=P(n-2)+P(n-3)
```

The integer spiral begins 1,1,1,2,2,3,4,5,7,9,12,16,21. In axial coordinates with basis (1,0) and (1/2,√3/2), cycle directions (1,0),(0,1),(-1,1),(-1,0),(0,-1),(1,-1). Starting V0=0, use V(n+1)=Vn+Pn d_n and triangle [Vn,V(n+1),Vn+Pn d_(n+1)]. [1]

```
u=exp(i*pi/3);    q=rho*u;    v_n=M*q^n/(q-1)

Delta_n=[v_n, v_n+M*q^n, v_n+M*u*q^n]
```

For the limit, multiplication by q sends each triangle to the next. Since |q|=ρ and u has angle 60°, each triangle is equilateral with side $M\rho^n$. With vertices A_n, B_n, C_n in the displayed order, the shared-edge identities are C_(n-1)=A_(n-5) and C_n=B_(n-5). Thus the new attachment edge A_nC_n is the union of B_(n-1)C_(n-1) and A_(n-5)B_(n-5), with lengths T_(n-1)+T_(n-5)=T_n.

The new validator reduces their axial coordinates in the exact algebraic field Q(ρ), using $\rho^3=\rho+1$, and confirms the shared endpoints. It also uses an exact separating-axis test for all 253 pairs in the original 23-triangle limit window and all 136 pairs in the original 17-triangle integer window. This is stronger than the original floating-point limit-overlap check.

The root is independently bracketed by 280 rational bisections of x^3-x-1 between 1 and 2. Ordering in Q(ρ) uses that rational isolating interval. The review script also tests corrupted SVGs with one-unit changes and coincident triangles; these negative controls must fail the relevant geometry checks.

The plastic number is irrational, so no finite ratio of integer Padovan terms equals it. A finite ruler-and-compass construction from a rational unit cannot produce it either: its irreducible cubic has degree 3. The exact limit is specified algebraically; the export is numerical. The literature distinguishes triangle growth and the plastic limit. [2]

This connected Padovan spiral has edge directions separated by 60°. It cannot supply both perpendicular edge directions of this square logo by merely tracing its triangle edges. Length transfers plus explicitly constructed perpendiculars are what this design uses. No stronger all-boundaries-on-one-spiral claim is made.

08 Reproduction and provenance

The delivered logo remains byte-for-byte unchanged. The corrected record and overlays supply the missing demonstration.

The baseline ZIP in review/baseline-v1.zip preserves the reviewed work. Its SHA-256 is 777340a90bbeb2209956d77f2ef0e25c0a8d38a007e73df326abe540b4a68a2c. The JSON audit includes the baseline SVG hash and each result. The revised package has a fresh checksum file.

Files. HORTRAME-logo.svg is the audited vector. HORTRAME-connected-spiral.svg shows the fixed source. HORTRAME-transparent-construction.svg shows the copied lengths with transparent overlaps. Each overlay also has PDF and PNG versions. review/overlay-transforms.json records every copy matrix. review/adversarial-review.json records the fresh audit.

Rebuild. Use Python 3 in a virtual environment, install requirements.txt, and put Poppler's pdftocairo on PATH. DejaVu Sans fonts are required for these review layouts. The root build.py recreates the base artwork, runs the fresh audit, then generates this revised record and both overlays. The historical generator is retained in review/build_v1.py, with only its output directory adjusted; its original bytes remain in the baseline ZIP.

```
.venv/bin/python -m pip install -r requirements.txt

.venv/bin/python build.py
```

The two overlay SVGs expose their source indices and transformation matrices. Their PDF and PNG versions are viewing exports. The clean logo files remain separate so construction geometry cannot accidentally become part of the mark.

Record and overlay styling. A4 record with 44-point margins; DejaVu Sans text and monospace equations. Overlay colors identify M (purple), S (blue), G (green), W (orange); fills use partial opacity. The same world coordinates are uniformly fitted to each output box. Camera fit is not a change to the triangles' relative scales. Overlay SVG groups retain the individual rigid-transform matrices for inspection.

Sources. [1] Robert Dickau, Padovan and Fibonacci Spirals, original geometric exposition: robertdickau.com/padovan-fibonacci-spirals.html. [2] V. W. de Spinadel and A. Redondo Buitrago, Towards van der Laan's Plastic Number in the Plane, Journal for Geometry and Graphics 13(2), 163-175 (2009): heldermann-verlag.de/jgg/jgg13/j13h2spin.pdf. Sources support the mathematical background, not the aesthetic choices for this logo.