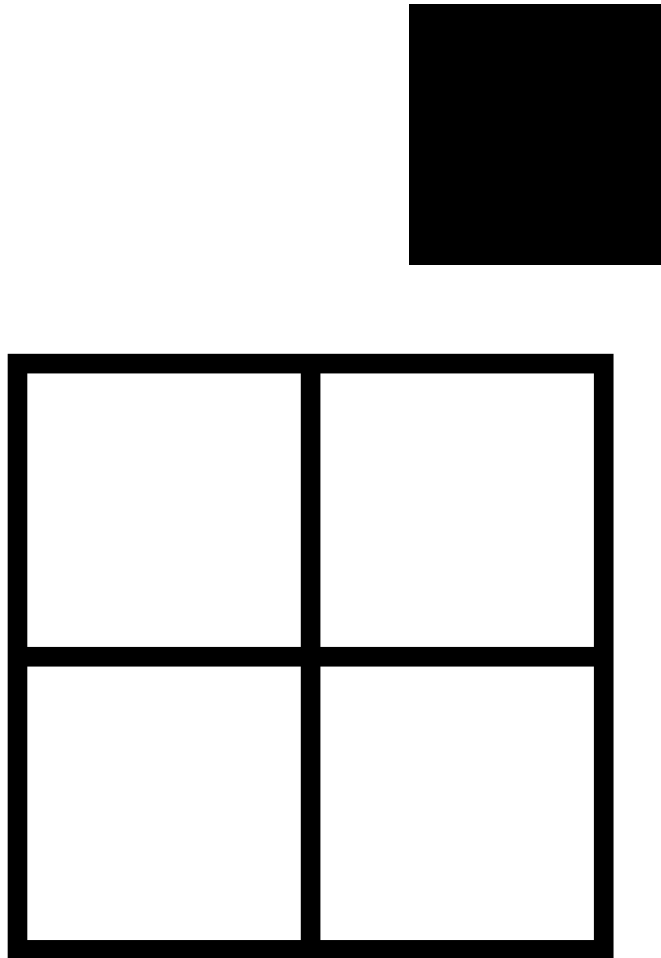


01 The new mark

The reference supplies the idea. One connected spiral now fixes the sizes and placement of both squares.

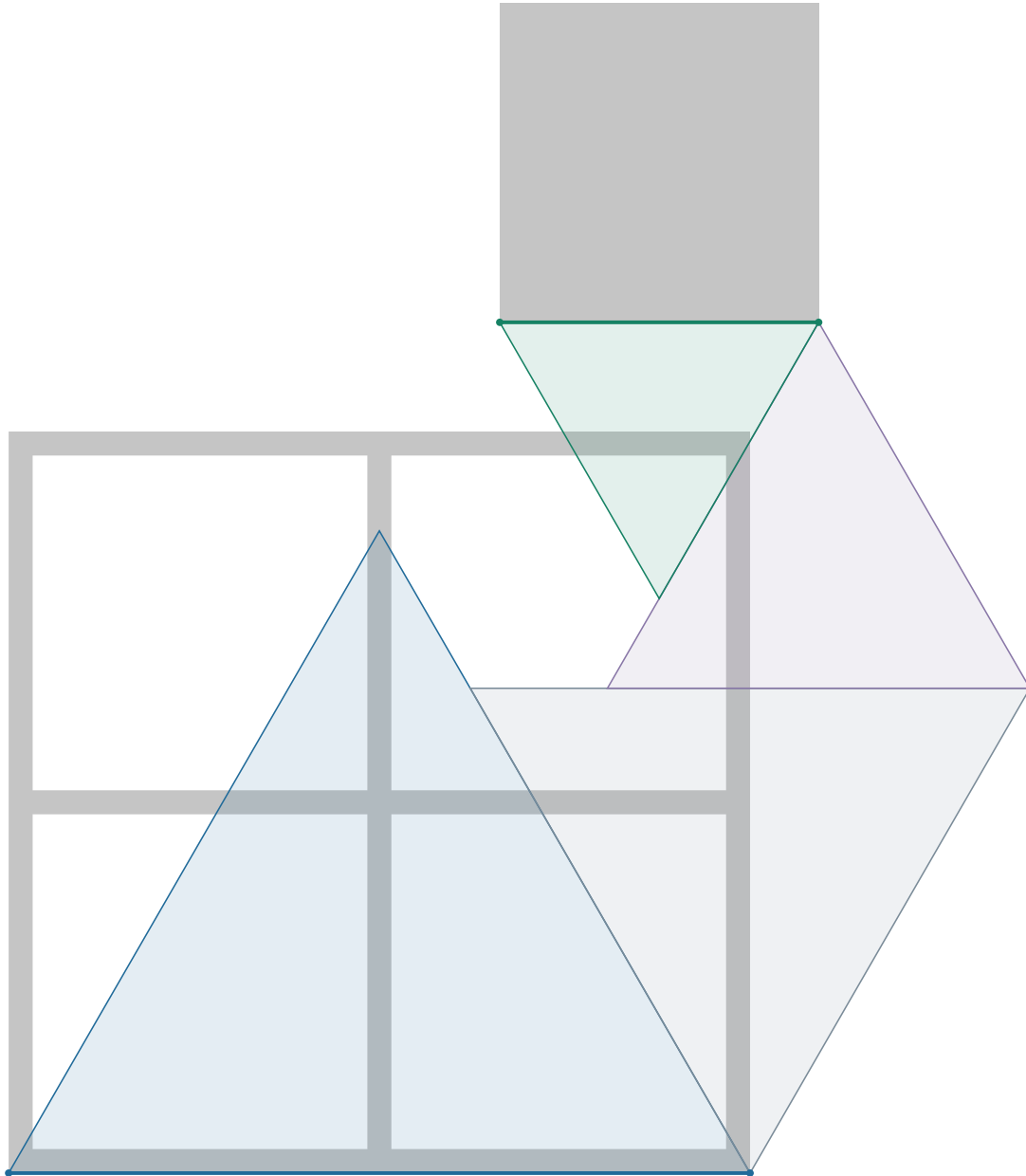


The large square is erected on the horizontal edge of T0. The detached square is erected on the horizontal edge of T-3 in the same connected Padovan-limit spiral. It is the first smaller candidate whose whole square lies above the large square. The gap and overhang are consequences of those edges.

Uniform border thickness follows a second explicit constraint: the four-cell frame and the solid square contain equal black area. This replaces the previous independently selected spacing and thickness relationships.

02 The connected construction

Every triangle stays in place. The highlighted edges coincide with actual edges of the two squares.



Blue T0 shares the main square's entire bottom edge. Green T-3 shares the detached square's entire bottom edge. The intermediate triangles stay connected to both. No triangle is copied onto another location or resized to fit a feature.

The drawing shows only four consecutive triangles, T0 through T-3. Their largest side is D . This is a finite window of the exact self-similar limit, not four integer Padovan terms.

03 Why this square, in this position

The selection rule is geometric separation, not resemblance to the old dimensions.

Work in coordinates with the main square $[0,D] \times [0,D]$, y downward. For each triangle encountered inward from T0, erect a square upward on its horizontal side. T-1 and T-2 both overlap the main square. T-3 is the first whose supporting edge is strictly above $y=0$.

Candidate	Side / D	Bottom y / D	Decision
T-1	0.754878	0.346257	Overlaps main
T-2	0.569840	0.346257	Overlaps main
T-3	0.430160	-0.147239	First detached

$$\rho^3 = \rho + 1; \quad h = \sqrt{3}/2$$

$$S = D/\rho^3$$

$$G = D*(h*\rho - 1)$$

$$O = (D/2)*(\rho^{-1} - \rho^{-2})$$

Here S is the detached-square side, G its clear vertical gap and O its right overhang. These are derived coordinates of the selected edge. There is no separate adjustment of G or O.

To obtain G, add the vertical components of the two preceding sloping steps. Their total rise is $hD(\rho^{-1} + \rho^{-2}) = hD\rho$. The main square rises D from its own supporting edge, leaving $G = hD\rho - D$. The horizontal projection gives $O = D(\rho^{-1} - \rho^{-2})/2$.

The square occupies $[D+O-S, D+O] \times [-G-S, -G]$. The choice to build both squares upward, and to put the detached one at upper right, remains a declared design choice. Once those rules and D are fixed, S, G and O have no remaining independent freedom.

04 Border weight and exact dimensions

The hollow component and solid component have equal black area. Four apertures remain congruent.

Let W be the width of every border and divider, and C the side of each clear opening. Two openings and three bars span the outer width: $2C+3W=D$. The grid's black area is D^2-4C^2 ; the detached square's area is S^2 . Requiring equality gives a unique positive W for the selected S .

$$C = \text{sqrt}(D^2-S^2)/2$$
$$W = (D-\text{sqrt}(D^2-S^2))/3$$
$$D^2 - 4*C^2 = S^2$$

Dimension	D=1000 units
Detached-square side S	430.159709002
Clear gap G	147.239403823
Right overhang O	92.518687624
Uniform thickness W	32.415732070
Aperture side C	451.376401895
Black area of each component	185037.375249

The frame is the outer D -by- D square with the four products $l_i \times l_j$ removed, where $l_0=(W,W+C)$ and $l_1=(2W+C,2W+2C)$. This centres the dividers and gives four identical openings. All visible angles are 90° .

Equal black area is a geometric balance criterion chosen for this redesign. It is not a theorem about perceived visual weight. Unlike selecting another distant spiral rung for line thickness, it ties W directly to both visible components.

Appearance: pure black fills, transparent master background, sharp corners, no lettering or effects. The master uses a compound even-odd path plus one rectangle. Clear space is G around the full ink bounds. No pixel snapping or optical shift is applied.

05 Every decision has a rule

Overall scale is the only adjustable measurement. The rules below determine every visible dimension.

Starting idea: a square grid and one detached square

These are the two forms retained from the inspiration. Four cells are the smallest nontrivial square array subdivided in both directions: 2×2 . No proportions are measured from the reference.

One connected spiral supplies both supporting edges

Equilateral triangles supply 60° turns; the user-requested plastic limit supplies the scale ratio p . Erecting squares perpendicular to horizontal triangle edges supplies 90° corners. One global orientation puts the detached component at upper right, as in the inspiration.

Choose the largest smaller square that is fully above the grid

Moving inward tests smaller candidates in descending size. The first two intersect the main square; the third clears it. This maximal detached choice selects T-3 and fixes S, G and O together. The index, gap and overhang are not fitted separately.

Make the four-cell structure symmetric and uniform

Four congruent square openings preserve the main square's rotations and reflections. A single common width for the outer frame and dividers gives $2C+3W=D$. This removes separate horizontal, vertical, inner and outer weight adjustments.

Give the two components equal geometric weight

Define weight here as black area. The equality $D^2-4C^2=S^2$, combined with the span equation, uniquely fixes C and W. The 1:1 area rule gives both components equal ink area; it does not assert equal perceived weight.

Reuse the derived gap for clear space

Set the minimum export margin to G on each side of the ink bounds, repeating the existing separation instead of adding another length. Black fill follows the reference. True squares require straight edges and sharp corners; no rounding or optical offsets are introduced.

These are justified design axioms, followed by derived measurements. Mathematics does not uniquely choose the axioms. With them fixed, the drawing has no independent size, spacing, thickness or alignment controls beyond its overall scale.

06 Definition, verification and limits

The geometry is specified algebraically and checked against the exported vector artwork.

```
u=exp(i*pi/3); q=rho*u; v_n=D*q^n/(q-1)
```

```
Triangle n: [v_n, v_n+D*q^n, v_n+D*u*q^n]
```

Apply one global mapping $z_{\text{screen}}=(D+iD)-(z-v_0)$ to every vertex. The resulting coordinates are interpreted in SVG's downward-y plane. It places T0's horizontal side on the main square's bottom edge and puts the selected detached square at upper right. There are no per-triangle transformations.

The model uses the plastic limit of the equilateral-triangle Padovan spiral. Integer Padovan terms obey $P_0=P_1=P_2=1$ and $P_n=P(n-2)+P(n-3)$; their finite ratios do not equal the irrational p . The side lengths $L_n=Dp^n$ obey the same recurrence. The equations, global mapping and candidate sequence completely specify the displayed construction. [1,2]

Validation checks the cubic, equilateral triangles, tile non-overlap, candidate selection, supporting edges, grid and equal area. A separate audit.py uses bisection and real-coordinate steps without importing the builder. It reads the exported SVG and also checks the displayed Padovan length identity $L_0=L-2+L-3$. Its largest vertex discrepancy is below 10^{-12} units at $D=1000$.

A finite straightedge-and-compass construction from a rational unit cannot generate p , whose minimal polynomial has degree 3. The exact geometry is algebraically defined; SVG numbers and PNG pixels are approximations. No finite rendering is literally infinitely precise. The artwork is uniquely specified once the stated design rules are accepted. Those rules themselves are not forced by mathematics.

Rebuild inside a Python virtual environment with reportlab installed, DejaVu Sans fonts and Poppler's pdftocairo available. Run build.py; it also runs audit.py. geometry.json gives the model. validation.json and independent-audit.json record the checks. The previous revision is retained in source/previous-r2.zip for provenance.

```
.venv/bin/python -m pip install -r requirements.txt
```

```
.venv/bin/python build.py
```

[1] Robert Dickau, Padovan and Fibonacci Spirals, robertdickau.com/padovan-fibonacci-spirals.html. [2] V. W. de Spinadel and A. Redondo Buitrago, Towards van der Laan's Plastic Number in the Plane, Journal for Geometry and Graphics 13(2), 163-175 (2009), heldermann-verlag.de/jgg/jgg13/j13h2spin.pdf. These sources support the spiral and limit mathematics; this logo selection and equal-area rule are new design choices in this record.