

Translation-Equivariant Disturbance Recovery in Delayed Open Priority Networks

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Abstract—We study recovery of relative three-dimensional geometry without an absolute restoring force on the formation translation. A least-squares translation removes the mean target error. For equal-gain double integrators, the resulting common mode is a damped integrator and the relative modes are stable second-order systems. A causal realization uses complete same-time origin snapshots, a shared commit clock and a current local residual innovation. Its estimation errors sum to zero exactly, although individual estimates are stale. A sufficient small-age condition proves boundedness and recovery for the coupled sampled feedback under actual communication contracts, including directional and state-dependent delays. An actual delayed priority process supplies convergent inputs under bounded use ages and joint connectivity of consumed influence. Finite cohort changes retain surviving physical and estimator states and preserve departed bodies under a separate law. Shared anchor tables cancel from terminal relative shape; fixed local profile tables yield an explicit priority-dependent shape and a nonconstancy criterion. Finite-variation velocity impulses, conditional active-body tubes and an exact nominal event map are treated separately. Finite checks illustrate the formulas. No rigid-body implementation, continuous trajectory safety or novelty claim is inferred from them.

Index Terms—formation control, communication delay, consensus, disturbance recovery, open multi-agent systems.

I. MODEL, TARGET AND CLAIM BOUNDARIES

Fix a nonempty active cohort I of size n and an ordered objective set of size m . Physical vectors are in $\mathbb{R}^3$; n, m and physical dimension are independent. Agent i has position x_i , velocity v_i , fixed offset r_i , and probability row w_i . Initially use one fixed $m \times 3$ anchor matrix Z and set $q_i = w_i Z$. Section VIII explicitly adds fixed local matrices Z_i . A bar denotes the unweighted current-cohort mean. Put

$$p_i = x_i - r_i, \quad h_i = p_i - q_i, \quad \xi = \bar{p}, \quad \eta_i = p_i - \xi. \quad (1)$$

Thus $\sum_i \eta_i = 0$. Equal gains $a, b > 0$ give $K = ab$, $C = a + b$. Norms are Euclidean unless another norm is specified. Ordinary disturbances are locally integrable; goal vectors are locally bounded and measurable. Positions are continuously differentiable and velocities locally absolutely continuous in the ordinary model. Differential equations hold almost everywhere. Section IV supplies a separate integral solution class for impulses.

The target is relative recovery for every actual execution satisfying the declared support, information, timing and gain hypotheses. A finite common position endpoint requires an additional mean-forcing condition. We distinguish exact optimizer identities, causal constructions, dynamical guarantees,

conditional physical consequences and finite numerical observations. A clock, correct cohort service, complete origin information and exact arithmetic are counted resources. A result for this model does not establish those resources in an existing simulator or vehicle. Literature integration is owner-supplied and pending; no priority-of-publication claim is made.

II. TRANSLATION OPTIMUM AND MODAL DYNAMICS

Proposition 1 (A1–A3: translation optimum). *At each time the function*

$$J_t(s) = \frac{1}{2n} \sum_i \|h_i(t) - s\|^2, \quad s \in \mathbb{R}^3, \quad (2)$$

has the unique minimizer $s^* = \bar{h} = \bar{p} - \bar{q}$. With targets $c_i = r_i + q_i + s^*$, one has $\bar{c} - \bar{r} = \bar{p}$. A common displacement Δ of current positions changes s^* and c_i by Δ and leaves $c_i - x_i$ unchanged.

Proof. The zero-sum identity $\sum_i (h_i - \bar{h}) = 0$ gives

$$J_t(s) = J_t(\bar{h}) + \frac{1}{2} \|s - \bar{h}\|^2. \quad (3)$$

This proves existence and uniqueness without an anchor-rank condition. Averaging the targets proves the mean identity. Adding Δ to every h_i proves the displacement identities. For $n = 1$, $c_1 = x_1$ for every goal and offset. $\square$

The instantaneous identities do not assert equivariance of an untranslated radio environment or a complete event history. A fixed absolute target fails the error identity: $c_i^0 - (x_i + \Delta) = (c_i^0 - x_i) - \Delta$. Substituting $\bar{x}$ for s^* changes the target mean by $\bar{r} + \bar{q}$. For example $n = m = 1$, $x = 0$, $r = q = e = (1, 0, 0)$ gives correct target zero and substituted target $2e$.

Replacing fixed anchors with current positions also changes the model. With $n = m = 2$, $r_1 = r_2 = 0$, $(x_1, x_2) = (0, 2e)$ and rows $(3/4, 1/4), (1/4, 3/4)$, fixed zero anchors give targets (e, e) . Position-valued anchors give $q_1 = e/2, q_2 = 3e/2$, $s^* = 0$ and targets $(e/2, 3e/2)$. Both computations use positive rows.

Use the nominal plant

$$\dot{x}_i = v_i, \quad \dot{v}_i = K(c_i - x_i) - C v_i + d_i. \quad (4)$$

Theorem 1 (B1–B3: common and relative modes). *For the exact optimizer targets,*

$$\ddot{\xi} + C\dot{\xi} = \bar{d}, \quad (5)$$

$$\ddot{\eta}_i + C\dot{\eta}_i + K\eta_i = K(q_i - \bar{q}) + (d_i - \bar{d}). \quad (6)$$

If $q_i - \bar{q} \rightarrow 0$ and $d_i - \bar{d} \rightarrow 0$, then $\eta_i, \dot{\eta}_i \rightarrow 0$. If additionally $\bar{d} \in L^1([T, \infty); \mathbb{R}^3)$, then $\dot{\xi} \rightarrow 0$ and

$$\xi_\infty = \xi(T) + \frac{\dot{\xi}(T)}{C} + \frac{1}{C} \int_T^\infty \bar{d}(s) ds. \quad (7)$$

Proof. The restoring error is $c_i - x_i = q_i - \bar{q} - \eta_i$ and has zero mean. Averaging and subtracting proves the equations; common gains are used in this cancellation. The position impulse response of $(D + a)(D + b)$ is

$$g(t) = \begin{cases} (e^{-at} - e^{-bt})/(b - a), & a \neq b, \\ te^{-at}, & a = b, \end{cases} \quad t \geq 0. \quad (8)$$

It satisfies $g(0) = 0$, $g'(0) = 1$, $g \geq 0$ and $\int_0^\infty g = 1/K$. For initial position y_0 and velocity z_0 , the homogeneous position is

$$y_h(t) = \begin{cases} \frac{(z_0 + by_0)e^{-at} - (z_0 + ay_0)e^{-bt}}{b - a}, & a \neq b, \\ [y_0 + (z_0 + ay_0)t]e^{-at}, & a = b. \end{cases} \quad (9)$$

Both y_h and $z_h = y'_h$ decay. Relative variation of constants gives $\eta_i = y_h + g * f_i$ and $\dot{\eta}_i = z_h + g' * f_i$, where f_i is the right side of (6). A finite locally integrable prefix contributes a decaying convolution. On a sufficiently late tail $\|f_i\|$ is arbitrarily small, so the two tail convolutions are bounded by that small number times $\|g\|_{L^1}$ and $\|g'\|_{L^1}$. This proves recovery.

The mean velocity is

$$\dot{\xi}(t) = e^{-C(t-T)}\dot{\xi}(T) + \int_T^t e^{-C(t-s)}\bar{d}(s) ds. \quad (10)$$

Integrating once gives

$$\begin{aligned} \xi(t) &= \xi(T) + \frac{1 - e^{-C(t-T)}}{C} \dot{\xi}(T) \\ &\quad + \frac{1}{C} \int_T^t (1 - e^{-C(t-s)}) \bar{d}(s) ds. \end{aligned} \quad (11)$$

Dominated convergence under the L^1 premise proves (7); the prefix/tail argument proves velocity decay. $\square$

If the mean force is zero after T , the integral in (7) vanishes. Merely vanishing mean force is insufficient: $\xi(t) = \log(1+t)e$ solves (5) with $\bar{d} = [C/(1+t) - 1/(1+t)^2]e \rightarrow 0$, yet has no finite endpoint. Constant nonzero mean force gives asymptotic velocity $\bar{d}/C$. Unequal gains need not preserve zero mean restoring force.

Shared anchors have an additional limitation. When $w_i \rightarrow \mu$, every $q_i \rightarrow \mu Z$ and $q_i - \bar{q} \rightarrow 0$. Terminal pair geometry is $r_i - r_j$; the common priority image cancels. Priority-dependent terminal geometry requires the explicit extension in Section VIII.

III. IMPERFECT ESTIMATION AND INPUT BOUNDS

Let $\hat{s}_i = s^* + \epsilon_i$ and use $c_i = r_i + q_i + \hat{s}_i$. Averaging and subtracting (4) gives the exact bridge

$$\ddot{\xi} + C\dot{\xi} = K\bar{e} + \bar{d}, \quad (12)$$

$$\ddot{\eta}_i + C\dot{\eta}_i + K\eta_i = F_i, \quad (13)$$

$$F_i = K(q_i - \bar{q}) + K(\epsilon_i - \bar{e}) + (d_i - \bar{d}).$$

These identities do not give a node access to ϵ_i . Put

$$G_x(\lambda) = \int_0^\infty e^{\lambda t} g(t) dt = \frac{1}{(a - \lambda)(b - \lambda)}, \quad (14)$$

$$G_v(\lambda) = \int_0^\infty e^{\lambda t} |g'(t)| dt, \quad 0 \leq \lambda < \min(a, b). \quad (15)$$

Both are finite. Direct differentiation shows that g' changes sign once, at $\log(b/a)/(b - a)$ or $1/a$. Consequently

$$G_v(0) = 2 \max_{t \geq 0} g(t) = \begin{cases} \frac{2}{b} (a/b)^{a/(b-a)}, & a \neq b, \\ 2/(ae), & a = b. \end{cases} \quad (16)$$

For bounded F_i , the convolution triangle inequality gives

$$\|\eta_i(t)\| \leq \|y_h(t)\| + \|F_i\|_\infty / K, \quad (17)$$

$$\|\dot{\eta}_i(t)\| \leq \|z_h(t)\| + G_v(0) \|F_i\|_\infty. \quad (18)$$

If Q, E, D bound the respective limiting upper norms of the priority, differential estimation and differential disturbance terms, prefix/tail splitting gives

$$\limsup \|\eta_i\| \leq Q + E + D/K, \quad (19)$$

$$\limsup \|\dot{\eta}_i\| \leq G_v(0)(KQ + KE + D). \quad (20)$$

Vanishing F_i gives exact relative recovery. Mean integrability must be imposed on the complete effective force $K\bar{e} + \bar{d}$; its estimator contribution to the endpoint is $(K/C) \int \bar{e}$. A bounded mean force bounds mean velocity but need not bound mean position.

If $q_i - \bar{q} \rightarrow 0$, $\epsilon_i \rightarrow \beta_i$ and $d_i \rightarrow D_i$, subtracting the constant relative equilibrium gives

$$\eta_i \rightarrow \beta_i - \bar{\beta} + (D_i - \bar{D})/K, \quad \dot{\eta}_i \rightarrow 0. \quad (21)$$

A nonzero limiting common force gives nonzero limiting mean velocity. Zero limiting common force still does not imply integrability.

For $\|F_i(t)\| \leq Me^{-\rho t}$ and $0 < \lambda < \min(a, b, \rho)$, multiplying the convolutions by $e^{\lambda t}$ proves bounds

$$e^{\lambda t} \|\eta_i(t)\| \leq H_x(\lambda) + MG_x(\lambda), \quad (22)$$

$$e^{\lambda t} \|\dot{\eta}_i(t)\| \leq H_v(\lambda) + MG_v(\lambda), \quad (23)$$

where H_x, H_v are the finite weighted suprema of the homogeneous responses. Equal decay rates can produce polynomial factors; the strict inequalities are part of the claim.

For a held estimator, require a complete partition $0 = t_0 < t_1 < \dots \rightarrow \infty$ with gaps at most H . If $\hat{s}_i = s^*(t_k) + \delta_i(k)$ on $[t_k, t_{k+1})$, then $\epsilon_i = \delta_i(k) + s^*(t_k) - s^*(t)$. Only when s^* is locally absolutely continuous may one write

$$\|\epsilon_i(t)\| \leq \|\delta_i(k)\| + \int_{t_k}^t \|\dot{s}^*(u)\| du. \quad (24)$$

A global derivative bound gives $\delta + MH$; a vanishing derivative and vanishing sample error give vanishing estimation error. If the derivative and held error are L^1 , Fubini bounds the integrated staleness by $H \|\dot{s}^*\|_{L^1}$. This does not cover discontinuous priority signals by differentiation. Bounded gaps alone do not define an infinite-time clock: $t_k = 1 - 2^{-k}$ accumulates at one.

IV. IMPULSES AND THE INITIAL CUT

With additional vector Radon measures M_i , use continuous locally absolutely continuous positions and right-continuous locally bounded-variation velocities. At an atom J_i at T , $v_i(T+) = v_i(T-) + J_i$. For $t \geq T$, the integral plant is

$$x_i(t) = x_i(T) + \int_T^t v_i(s) ds, \quad (25)$$

$$\begin{aligned} v_i(t) &= v_i(T+) \\ &+ \int_T^t [K(c_i - x_i) - Cv_i + d_{i,\text{reg}}](s) ds \\ &+ M_i((T, t]). \end{aligned} \quad (26)$$

An atom at T is already in the initial right-hand velocity. It is excluded from the future measure. The current-time atom is included in the right-continuous velocity. No classical position derivative at an impulse cut is required.

Let $f = K\bar{e} + \bar{d}_{\text{reg}}$, let F_i be (13) with regular disturbances, and put $\bar{M}_i = M_i - \bar{M}$. Variation of constants gives mean velocity

$$\begin{aligned} \bar{v}(t) &= e^{-C(t-T)}\bar{v}(T+) + \int_T^t e^{-C(t-s)}f(s) ds \\ &+ \int_{(T,t]} e^{-C(t-s)} d\bar{M}(s). \end{aligned} \quad (27)$$

The mean position uses kernel $(1 - e^{-C(t-s)})/C$ against both forces. The relative position and velocity use g and g' against $F_i(s) ds$ and $dM_i(s)$, with homogeneous data at $(\eta_i(T), v_i(T+) - \bar{v}(T+))$. Since $g(0) = 0$ and $g'(0) = 1$, position is continuous and velocity has the prescribed jump.

Proposition 2 (Measure recovery and endpoint). *If F_i tends to zero on a late tail and $\bar{M}_i$ has finite future total variation, relative position and velocity tend to zero. If $f \in L^1([T, \infty))$ and $\bar{M}$ has finite future total variation, then mean velocity tends to zero and*

$$\xi_\infty = \xi(T) + \frac{\bar{v}(T+)}{C} + \frac{1}{C} \left[\int_T^\infty f(s) ds + \bar{M}((T, \infty)) \right]. \quad (28)$$

Proof. Split each measure convolution into a compact prefix and a tail of arbitrarily small total variation. The bounded decaying kernels make the prefix vanish and bound the tail by its variation times the kernel supremum. The ordinary convolution uses the earlier prefix/tail argument. Dominated convergence applies to the bounded mean position kernel under the stated L^1 and total-variation premises; the mean velocity kernels decay and use the same split. $\square$

Finite impulse variation does not supply vanishing regular relative forcing or integrable regular common forcing. With $a = b = 1$, one body at $x(T) = 0$, $v(T-) = 0$ and atom e at T has endpoint $e/2$. Using both $v(T+) = e$ and a closed future measure that includes the same atom falsely gives endpoint e . Either right/open or left/closed conventions work; they cannot be mixed.

V. CAUSAL SAME-TIME RESIDUAL SNAPSHOTS

The following service is an added construction. Correct incarnations know one ordered cohort record, its objective metadata and a common spatial coordinate convention. Local feedback is exact. A shared sampling and commit clock is available. Choose $h > 0$, $0 \leq L < h$, sample at $t_k = kh$ and commit epoch k at $t_k + L$. Simultaneous primitives have a declared internal order and one common pre/post-event sampling convention. Every epoch's complete frames arrive at every continuing cohort member by its commit. Deliveries may be asynchronous, directional, nonuniform and causally state-dependent; commit use is coherent.

A sufficient delivery contract has strongly connected unions of locally finite forwarding opportunities in every interval $(u, u + B]$. A sender possessing a frame before an opportunity forwards it, with bounded transmission plus processing $\delta + Q_s$. Correct immutable delivery and queue bounds include the actual offered load. Then

$$L = (n - 1)(B + \delta + Q_s) \quad (29)$$

is sufficient; a singleton uses $L = 0$. Indeed, from an informed proper set at time u , the next opportunity union contains an edge to its complement. That frame reaches a new node by $u + B + \delta + Q_s$. Repeating at most $n - 1$ times proves the bound. The open-left window and declared simultaneous ordering ensure that an initially informed sender possesses the retained frame before the selected opportunity. A configured graph alone is insufficient.

The forwarding cadence B is distinct from the sampling period h . Feasibility requires choosing them so that $L < h$. Taking $B = h$ in a multihop version of (29) need not be feasible. Direct delivery permits a sharper one-hop contract.

At t_k , origin j samples only its own actual

$$H_{j,k} = x_j(t_k) - r_j - q_j(t_k). \quad (30)$$

The frame binds cohort prefix, ordered support and metadata, source incarnation, epoch, generation time and this vector. Forwarders retain the immutable origin values. Wrong destination, incarnation or cohort copies are excluded. Identical duplicates add no second term; conflicting identities/payloads are outside the compliant model. Future epochs wait, and old epochs cannot replace a newer committed table. Each node keeps a committed origin table and a pending epoch table. Because $L < h$, at most one sampling epoch is pending. At commit, every node computes in the declared label order

$$m_{i,k} = H_{i,k} - \frac{1}{n} \sum_j H_{j,k}. \quad (31)$$

The arithmetic mean is calculated locally from received complete origin information. It is not supplied as an uncensored centroid oracle and is not a conserved packet mass.

The table requires $O(n)$ vectors per node and $O(n^2)$ network-wide, plus cohort, control and two-epoch metadata. Epoch counters use growing $O(\log k)$ bits; exact real coordinates do not imply finite-bit storage. Let F_k count actual directed forwarding opportunities carrying epoch- k traffic, including repeated opportunities on the same edge. Forwarding

all n origins at each opportunity costs at most nF_k individual frame transmissions, or at most F_k bundles containing up to n frames. No distinct-edge bound substitutes for F_k without an explicit opportunity cap or a specified send-once policy. Priority and control traffic are additional. Table storage excludes in-flight duplicates, pending delivery queues and total processing work; these need their own workload and buffer bounds. Locally finite traffic need not have a uniform peak bound. These are sufficient resource counts, not minimum costs.

Between commits the local innovation decoder is

$$\hat{s}_i(t) = h_i(t) - m_i, \quad c_i(t) = r_i + q_i(t) + \hat{s}_i(t) = x_i(t) - m_i. \quad (32)$$

Before coherent bootstrap a declared finite waiting law is used, without an arithmetic error guarantee. Let $\sigma(t) = t_k$ on $[t_k + L, t_{k+1} + L)$ and $H = h + L$, so $0 \leq t - \sigma(t) < H$. Write $e_i = h_i - \bar{h}$.

Theorem 2 (D2: centered snapshot identity). *At coherent fixed-support use times,*

$$m_i = e_i(\sigma(t)), \quad \epsilon_i = e_i(t) - e_i(\sigma(t)), \quad \sum_i \epsilon_i = 0. \quad (33)$$

Moreover the estimation error is bounded by the oscillation of e_i on $[t - H, t]$. If e_i is absolutely continuous there, it is bounded by $\int_{\sigma(t)}^t \|\dot{e}_i(u)\| du$.

Proof. The committed table contains exactly the actual same-time origin values. Centering therefore gives $m_i = e_i(\sigma)$. Substitute (32); the sum is zero at both the current and sampled times. The oscillation bound is immediate, and the derivative bound follows only under its absolute-continuity premise. $\square$

Thus a common unbounded drift can cancel even when individual residuals fail to converge. A common displacement with the committed table unchanged moves each decoded shift and target by that displacement and leaves the restoring error $-m_i$ unchanged. New same-time snapshots after a common displacement also have unchanged centered corrections.

Coherent commits are load-bearing. Two agents using respectively centered snapshots $(e, -e)$ and $(2e, -2e)$ have corrections $(e, -2e)$ with mean $-e/2$, adding common acceleration $Ke/2$. Asynchronous reception is permitted; different committed epochs do not preserve (33). Ordinary row-stochastic consensus also does not replace arithmetic collection. Sequential half-weight updates have period matrices

$$\begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}, \quad \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}, \quad (34)$$

with invariant rows $(2/3, 1/3)$ and $(1/3, 2/3)$; unequal residuals give different non-arithmetic limits.

VI. COUPLED DELAYED-FEEDBACK RECOVERY

The physical service clock in Section V starts sampling at zero and first commits at L . It is distinct from the analysis origin used for the following bounds. Choose an actual cut T_0 after coherent bootstrap, with all currently used residual tables coherent on fixed support and with a complete actual history on $[T_0 - H, T_0]$. Choose it late enough that this history

and every subsequently used residual source value have the fixed goal/support metadata under analysis. After membership changes, use the actual final-support cut of Section IX, enlarged if necessary to supply this whole history interval.

For the feedback equations, constants and bounds below, rebase physical time t_{phys} by $u = t_{\text{phys}} - T_0$ and then write t for u . Initial position, post-atom velocity, correction and history are their actual retained values at this cut; nothing is reset. The inherited sample clock has phase $\phi = (-T_0) \bmod h$, with samples $\phi + kh$ and commits $\phi + kh + L$ after integer reindexing. Only epochs actually generated in the execution are used, including valid past samples at negative rebased times. The sampled-time map is

$$\sigma_0(u) = \sigma_{\text{phys}}(T_0 + u) - T_0, \quad 0 \leq u - \sigma_0(u) < H. \quad (35)$$

Write σ for σ_0 in the following equations. The scored current bounds and rate estimates are for $u \geq 0$, with history on $[-H, 0]$; they do not bound an unrestricted pre-bootstrap waiting prefix. This is an analysis coordinate change, not a sampling restart, a change to stored packet epoch identities or a requirement that nodes detect T_0 . Physical event descriptions elsewhere retain their clock unless explicitly rebased. When applying a physical priority rate here, substitute $t_{\text{phys}} = T_0 + u$ and absorb the finite change of prefactor.

Write $\tilde{q}_i = q_i - \bar{q}$ and $\tilde{d}_i = d_i - \bar{d}$. Equations (4) and (33) give

$$\ddot{\xi} + C\dot{\xi} = \bar{d}, \quad (36)$$

$$\ddot{\eta}_i + C\dot{\eta}_i = -K\eta_i(\sigma(t)) + K\tilde{q}_i(\sigma(t)) + \tilde{d}_i(t). \quad (37)$$

Equivalently the stable undelayed relative operator is forced by $K[\eta_i(t) - \eta_i(\sigma(t))] + K\tilde{q}_i(\sigma(t)) + \tilde{d}_i(t)$. No derivative of a priority signal is used.

For an actual continuous initial position history on $[-H, 0]$ with bounded relative velocities, put

$$V_- = \sup_{[-H, 0]} \|\dot{\eta}_i\|, \quad H_v = \sup_{t \geq 0} \|z_h(t)\|, \quad (38)$$

$$H_x = \sup_{t \geq 0} \|y_h(t)\|.$$

Suppose $\|\tilde{q}_i\| \leq Q$, $\|\tilde{d}_i\| \leq D$. The shared-anchor hull diameter is a sufficient Q ; Section VIII uses the union of local profile rows.

Theorem 3 (E1–E2: small-age bound and recovery). *If $\kappa_0 = KHG_v(0) < 1$, every admissible fixed-support execution satisfies*

$$\sup_{t \geq -H} \|\dot{\eta}_i(t)\| \leq V := \frac{V_- + H_v + G_v(0)(KQ + D)}{1 - \kappa_0}, \quad (39)$$

$$\sup_{t \geq 0} \|\eta_i(t)\| \leq P := H_x + HV + Q + D/K. \quad (40)$$

If the differential goal and disturbance inputs tend to zero, then $\eta_i, \dot{\eta}_i, \epsilon_i \rightarrow 0$. A finite common translation follows only when the additional mean-forcing endpoint premises hold.

Proof. Continuity of positions gives

$$\|\eta_i(t) - \eta_i(\sigma(t))\| \leq H \sup_{-H \leq u \leq t} \|\dot{\eta}_i(u)\|. \quad (41)$$

On each finite horizon the method of steps, or locally finite sample/commit updates, gives finite states. Variation of constants bounds the current velocity supremum by $H_v + \kappa_0$ times the combined history/current supremum plus $G_v(0)(KQ + D)$. Taking its maximum with V_- and using $\max(A, B) \leq A + B$ proves (39). The bound is independent of the horizon. The position convolution and $G_x(0) = 1/K$ prove (40).

For recovery, absorb a finite locally integrable prefix into actual finite initial data and apply the bound to a late bounded tail. Let $V_\infty = \limsup_{t \rightarrow \infty} \|\dot{\eta}_i(t)\|$. Since $\sigma(t) \geq t - H$, sufficiently late delay errors are bounded by $H(V_\infty + \varepsilon)$. Finite-prefix convolution contributions and the vanishing exogenous tail inputs vanish. The velocity convolution therefore gives $V_\infty \leq \kappa_0(V_\infty + \varepsilon)$. Letting $\varepsilon \downarrow 0$ proves $V_\infty = 0$. Delay errors now vanish; the position convolution gives $\eta_i \rightarrow 0$. Finally $e_i = \eta_i - \tilde{q}_i \rightarrow 0$ at current and sampled times, proving $e_i \rightarrow 0$. $\square$

The condition is sufficient, not necessary or optimal. For $a = b$ it is $2aH/e < 1$. A singleton has zero relative state and requires no such inequality.

Proposition 3 (E3: strict exponential bound). *For $0 < \lambda < \min(a, b)$ require*

$$\kappa_\lambda = KHe^{\lambda H}G_v(\lambda) < 1. \quad (42)$$

Let $V_-^\lambda = \sup_{[-H, 0]} e^{\lambda s} \|\dot{\eta}_i(s)\|$, $Q_\lambda = \sup_{s \geq -H} e^{\lambda s} \|\tilde{q}_i(s)\|$ and $D_\lambda = \text{ess sup}_{t \geq 0} e^{\lambda t} \|\tilde{d}_i(t)\|$ be finite. Put $M_\lambda = Ke^{\lambda H}Q_\lambda + D_\lambda$. Then

$$\begin{aligned} V_\lambda &:= \sup_{t \geq -H} e^{\lambda t} \|\dot{\eta}_i(t)\|, \\ V_\lambda &\leq \frac{V_-^\lambda + H_v(\lambda) + G_v(\lambda)M_\lambda}{1 - \kappa_\lambda}, \end{aligned} \quad (43)$$

$$\begin{aligned} e^{\lambda t} \|\eta_i(t)\| &\leq H_x(\lambda) \\ &+ G_x(\lambda)[KHe^{\lambda H}V_\lambda + M_\lambda]. \end{aligned} \quad (44)$$

Proof. The weighted delay-position error is bounded by $He^{\lambda H}V_\lambda$. Sampled goal inputs gain the same factor $e^{\lambda H}$. Apply the weighted convolution estimates first on finite horizons and solve the same small-gain inequality. The resulting bound is horizon-independent. Continuity of $G_v(\lambda)$ near zero shows that a positive admissible λ exists whenever $\kappa_0 < 1$, provided the input decay supports that rate. $\square$

VII. PRIORITY CONVERGENCE ALONG ACTUAL SCHEDULES

After a causal final-support history cut, suppose the actual action-indexed priority law is

$$w_i(k+1) = a_{ii}(k)w_i(k) + \sum_{j \neq i} a_{ij}(k)w_j(k - \tau_{ij}(k)). \quad (45)$$

Each coefficient row is stochastic, $a_{ii} \geq \zeta > 0$, every positive off-diagonal coefficient is at least ζ , $\zeta < 1$, and each used age is at most D_p . The union of actual used influence in every B_p consecutive action indices is strongly connected. Inactive rows hold. Initial history rows are actual positive rows on final support. Packet availability is part of this assumption; arbitrary lag entries cannot invent packets.

Lemma 1 (P1: delayed priority contraction). *For $n \geq 2$, put*

$$L_p = (n-1)(B_p + D_p) + D_p, \quad \lambda_p = 1 - \zeta^{L_p}. \quad (46)$$

Every length- L_p augmented product has every entry of any chosen initial current-root column at least ζ^{L_p} . If o is the vector of initial augmented coordinate ranges, then

$$\|w_i(k) - \mu_\sigma\|_2 \leq \|o\|_2 \lambda_p^{\lfloor (k-K_0)/L_p \rfloor} \quad (47)$$

for one positive probability row μ_σ determined by the actual execution.

Proof. Augment by D_p layers of actual past rows. A current root coefficient persists via positive self-weights, with a lower bound ζ raised to elapsed action count. From an informed set at action u , wait D_p actions. Every read from that set in the next B_p actions then refers to an informed time. Strong connectivity supplies an edge to the complement, informing one further current node within $D_p + B_p$ actions. Repeat at most $n-1$ times and shift for another D_p actions to inform all history layers. Every factor is at least ζ , including unit shift factors. Subtract the common root column of mass ζ^{L_p} and divide the remainder by $1 - \zeta^{L_p}$ to obtain a stochastic map. Each coordinate range contracts by λ_p ; intervening stochastic maps cannot increase augmented ranges. Nested minima and maxima therefore converge to common coordinates. Positivity follows from the finite positive initial minima, and row sums remain one. The common endpoint lies between the current augmented extrema, proving (47). $\square$

A singleton holds its row; no zero-length block formula is applied. Action time is not physical time. If the locally finite embedding tends to infinite time and

$$k(t) - K_0 \geq \rho_{\min}(t - T) - C_p, \quad \rho_{\min} > 0, \quad (48)$$

then the contraction supplies physical decay rate

$$\nu_p = \rho_{\min}[-\log \lambda_p]/L_p. \quad (49)$$

Indeed the block factor is at most $\lambda_p^{-1-C_p/L_p} e^{-\nu_p(t-T)}$. An upper action-density bound alone does not supply this lower count or actual influence condition.

The lemma is pathwise for causal schedules, including those depending on motion and complete prior state. It does not give one parameter-independent endpoint functional across initial conditions whose realized schedules differ. Under shared anchors, it gives $\tilde{q}_i \rightarrow 0$; hence the causal estimator, relative dynamics and priority process converge together under their stated contracts. Uniform, directional unequal and bounded varying delays are included as subclasses of the actual use-age assumptions.

VIII. AN EXPLICIT PRIORITY-DEPENDENT SHAPE EXTENSION

Add fixed local profile matrices $Z_i \in \mathbb{R}^{m \times 3}$ in coherent cohort metadata, and set $q_i = w_i Z_i$. This changes the shared-anchor model. It has up to $3nm$ unique real template entries versus $3m$ in the shared definition. With only local profiles stored, each node stores $3m$ entries and the network stores $3nm$; service/log replicas are additional. No storage ratio

against an unspecified replication arrangement or resource minimum is claimed.

Let $\bar{Z} = n^{-1} \sum_i Z_i$. Every local goal and its mean belong to the hull of the union of all profile rows; its diameter bounds $\|\tilde{q}_i\|$ before the endpoint is known. For the priority endpoint of Lemma P1 define the analysis template

$$\rho_i = \mu_\sigma(Z_i - \bar{Z}), \quad \sum_i \rho_i = 0. \quad (50)$$

Executing nodes are not supplied ρ_i or a future priority endpoint.

Theorem 4 (H2: profile shape recovery). *Under the priority hypotheses, coherent snapshot service, $\kappa_0 < 1$ and vanishing differential ordinary disturbance, one has*

$$\eta_i \rightarrow \rho_i, \quad \dot{\eta}_i \rightarrow 0, \quad \epsilon_i \rightarrow 0. \quad (51)$$

The same conclusion holds with finite future differential impulse variation under the ordinary-forcing premises of Section X. When the mean endpoint premises hold,

$$x_{i,\infty} = r_i + \xi_\infty + \rho_i, \quad (52)$$

$$x_{i,\infty} - x_{j,\infty} = r_i - r_j + \mu_\sigma(Z_i - Z_j). \quad (53)$$

Proof. Continuity of the fixed linear maps gives $q_i \rightarrow \mu_\sigma Z_i$, hence $\tilde{q}_i \rightarrow \rho_i$. Put $\theta_i = \eta_i - \rho_i$. Subtracting the constant template in (37) gives

$$\begin{aligned} \ddot{\theta}_i + C\dot{\theta}_i &= -K\theta_i(\sigma(t)) \\ &+ K[\tilde{q}_i(\sigma(t)) - \rho_i] + \tilde{d}_i(t). \end{aligned} \quad (54)$$

The shifted input vanishes. The coupled recovery proof applies to actual shifted initial data. Since $e_i = \eta_i - \tilde{q}_i \rightarrow 0$, (33) gives estimation recovery. The zero mean error preserves (5); apply the conditional mean endpoint and substitute in (1). $\square$

For a physical rate, $C_Z = \max_i \|Z_i\|_{\text{op}}$ gives

$$\|\tilde{q}_i - \rho_i\| \leq 2C_Z \|o\|_2 \lambda_p^{\lfloor (k-K_0)/L_p \rfloor}. \quad (55)$$

Bound the projected local error and its mean separately to obtain this inequality. The weighted coupled estimate uses $\tilde{q}_i - \rho_i$ and shifted homogeneous initial data; the generally nonzero raw $\tilde{q}_i$ is not asserted to decay. The unshifted bounds (39)–(40) continue to use the union-profile diameter for availability before μ_σ is known.

Proposition 4 (H3: nonconstant shape criterion). *For $n \geq 2$, terminal pair geometry as a function of μ is nonconstant on the positive probability simplex exactly when some $Z_i - Z_j$ has two unequal objective rows.*

Proof. If all rows of each pair difference are equal, multiplication by any unit-sum row gives the same vector. Otherwise choose distinct rows a, b of one pair difference and positive rows $\mu_\pm = u \pm t(e_a - e_b)$, where u is uniform and $0 < t < 1/m$. Their pair geometries differ by $2t$ times the nonzero row difference. This proves nonconstancy of the simplex family, not reachability from one fixed initial history. Constant positive histories can realize either endpoint in the fixed-support initialization class. $\square$

A singleton has $\rho_1 = 0$. Shared $Z_i = Z$ recovers priority cancellation. With two objectives, $Z_1 = (0, e)$, $Z_2 = (0, -e)$ and offsets $(-3e, 3e)$, agreed rows $(3/4, 1/4)$ and $(1/4, 3/4)$ give pair displacements $-11e/2$ and $-9e/2$.

Corollary 1 (Common disturbances on an invariant template). *Suppose all priority rows and actually consumed priority data equal one fixed μ throughout the complete actual source/controller history at T_0 and the future execution. This history includes the generating times and priority values of every initial committed or pending residual epoch eligible for use. Suppose throughout that same controller history $\eta_i = \rho_i$ and relative velocity is zero. Under common ordinary forcing and common velocity impulses, an admissible coherent execution retains these relative values; $m_i = 0$ and the formation evolves only through the mean law.*

Proof. At the base cut, the valid committed table and every eligible pending epoch come from the stipulated actual history. At each such sample, $q_i = \mu Z_i$ and $h_i = \xi - \mu \bar{Z}$ is common, so the initial and later centered corrections are zero. Until the next sample the identical forcing and equal gains preserve identical velocities and fixed pair differences; a common impulse likewise preserves relative velocity. Induction over locally finite samples and impulses proves invariance. Current priority agreement alone does not imply that future stale priority data are also agreed; the complete-history premise is essential. $\square$

The strong complete-history corollary survived TESSERA's full-source R03 attack; this does not retrospectively include it in earlier module reviews. Current agreement alone cannot remove a correction generated by earlier disagreeing goals.

IX. FINITE COHORT CHANGES WITHOUT PHYSICAL DELETION

Use a finite body pool and distinct logical incarnation IDs. A correct service emits finitely many predecessor-linked records naming the next nonempty support, objective/profile/offset metadata and positive priority maps/bootstrap. Records activate in order; activation is announced far enough ahead for a declared complete receipt/application budget R . This counted sequencing/timing service is not Byzantine membership agreement.

Surviving bodies retain x_i, v_i, m_i . They do not reset the plant or erase their numerical correction at a record. A newborn body already exists at a finite physical state and waits under a specified velocity-damping law until coherent positive bootstrap; its new correction may be zero. A removed incarnation stops active priority and estimator operations, while its physical body remains under a declared retired law, for example $\dot{x} = v$, $\dot{v} = -Cv + d_{\text{ret}}$. Body reuse creates a new incarnation; old addressed packets are rejected.

Residual frames retain immutable cohort identity. Old-cohort frames cannot enter a new-cohort arithmetic calculation. Future-prefix frames wait; duplicates contribute once. These are snapshots rather than conserved estimator mass. During an incomplete transition a survivor may use retained

m_i with its current local residual, keeping $c_i = x_i - m_i$ well-defined. No exact new-cohort mean guarantee is asserted then. The next complete new-cohort snapshot performs the ordinary origin-table update; it does not globally zero states.

Admission changes the slot centroid to $(n\xi + p_{\text{new}})/(n+1)$. A departure for $n > 1$ changes it to $(n\xi - p_{\text{removed}})/(n-1)$. The mean goal can also change. An optimum may therefore jump while all surviving positions and velocities remain continuous. No cross-membership continuity or earlier-reference restoration is implied.

Theorem 5 (F1: eventual final-support recovery). *Under the finite record/bootstrap and eventual snapshot/priority/coupled contracts, a finite analysis cut exists after which every used residual snapshot and required priority history has final support. Applying the fixed-support results from the actual retained state/history proves final-cohort priority agreement, estimator recovery and the appropriate relative template recovery without a global numerical reset. A common endpoint retains its separate forcing conditions.*

Proof. Let T_f be the actual time by which the last record and bootstraps apply. Within $h + L$ a complete final-support residual snapshot commits. After another H , every sampled history used by the controller is final-support data; old-cohort packets are excluded, not numerically invented. Bounded generation/use ages and final record application yield a further priority cut whose entire D_p augmented history is actual final-support data. The embedding has no finite-time accumulation.

Locally finite prefix events, positive support maps, finite metadata and locally integrable disturbances keep states finite under the waiting and retained-correction linear laws. Choose T_0 still later if needed so its entire H -length actual physical history has final support and goal metadata, as required in Section VI. Its inherited sample phase and valid stored correction are retained. Starting analysis at this resulting history changes no state. Apply priority contraction, then coupled recovery to the shared or shifted profile input, and use the mean endpoint only after coherent final-support use. Transition errors are not hidden in a claimed zero mean invariant. $\square$

Before any information about a newborn residual arrives, an incumbent cannot compute the new arithmetic mean for every residual: indistinguishable incumbent observations with newborn residual zero or e give answers differing by $e/(n+1)$. This is a query-specific obstruction, not a minimum-resource theorem. Finite realized logs are additional storage, with no uniform bit budget absent a record-count/encoding bound.

X. IMPULSES IN THE COUPLED LOOP

Impulse event times t_ℓ and the cut T in this section are physical times, with $T = T_0$. When applying Section VI, rebase them to $u_\ell = t_\ell - T_0 > 0$ and use elapsed-time weights. The post-atom initial datum is unchanged; the physical future interval $(T_0, t_{\text{phys}}]$ becomes $(0, u]$, and the kernel age $t_{\text{phys}} - t_\ell = u - u_\ell$ is unchanged. Let a locally finite sequence of velocity jumps have differential impulses $\tilde{J}_{i,\ell} = J_{i,\ell} - \bar{J}_\ell$.

At an analysis cut T , use post-atom initial velocity and only $t_\ell > T$ as future impulses. Put

$$\Gamma_i = \sum_{t_\ell > T} \|\tilde{J}_{i,\ell}\|, \quad K_x = \sup_{t \geq 0} g(t), \quad K_v = \sup_{t \geq 0} |g'(t)|. \quad (56)$$

The relative convolution adds $\sum_{T < t_\ell \leq t} g(t - t_\ell) \tilde{J}_{i,\ell}$ for position and the analogous g' sum for velocity. For finite Γ_i , the coupled bounds become

$$V \leq \frac{V_- + H_v + G_v(0)(KQ + D) + K_v \Gamma_i}{1 - \kappa_0}, \quad (57)$$

$$P \leq H_x + HV + Q + D/K + K_x \Gamma_i. \quad (58)$$

Indeed the added convolutions are bounded by $K_x \Gamma_i$ and $K_v \Gamma_i$; position continuity still gives the integrated delay error. The finite-horizon supremum argument is otherwise unchanged. Finite impulse prefixes decay and finite-variation tails are arbitrarily small, so vanishing ordinary shifted differential inputs still imply recovery. Finite variation alone gives no exponential rate: require $\sum_{t_\ell > T} e^{\lambda(t_\ell - T)} \|\tilde{J}_{i,\ell}\| < \infty$ and the weighted kernel suprema, or a finite last impulse and exponential ordinary forcing.

The exact mean endpoint is (28) with $f = \bar{d}_{\text{reg}}$, because coherent errors sum to zero. Common impulse variation does not replace ordinary L^1 forcing. These hypotheses are on the declared translational model, not automatically on contacts, avoidance or rotorcraft attitude errors.

XI. TIMING, GEOMETRY AND THE NOMINAL IMPLEMENTATION

A. A sufficient noncircular transport model

The following relative-state, speed and radio envelopes start at the same actual coherent cut T_0 , with its retained-history constants and future-only impulse budgets. They supply no waiting-prefix guarantee. As an optional sufficient radio model, a signal expands from its generation position at speed C_{sig} in unobstructed space. If a receiver speed is bounded by $V_{\text{body}} < C_{\text{sig}}$, an initial source/receiver distance d_0 gives reception within $d_0/(C_{\text{sig}} - V_{\text{body}})$. The continuous function $\|x_i(g+s) - x_j(g)\| - C_{\text{sig}}s$ is nonnegative initially and at most $d_0 - (C_{\text{sig}} - V_{\text{body}})s$; it crosses zero by the stated bound.

For active pairs, the relative bound gives

$$\|x_i - x_j\| \leq \|r_i - r_j\| + 2P. \quad (59)$$

The mean law bounds speed by

$$\|\bar{v}(t)\| \leq \|\bar{v}(T+)\| + \|\bar{d}_{\text{reg}}\|_\infty / C + \sum_{t_\ell > T} \|\bar{J}_\ell\|. \quad (60)$$

Thus a sufficient V_{body} is this bound plus the largest relative V . Choose range covering selected edges and $C_{\text{sig}} > V_{\text{body}}$; then check the resulting delivery, service, forwarding, h, L, H and small-age inequalities jointly. Uniform hull bounds, not desired limiting priorities, enter P . A first-violation continuation argument preserves the declared distance/speed envelope with strict margins and coherent initial/service conditions: until a putative first violation the contracts provide the bounds; those bounds contradict crossing the strict envelope. Queue

service and packet correctness remain separate assumptions. This is a sufficient realization, not a conclusion from geometry alone. Other transport models may directly establish the actual contracts.

B. Exact nominal event map

Between correction, cohort and forcing events, m_i is constant. For piecewise constant d_i , put $U_i = -Km_i + d_i$. The active nominal equation is $\dot{v}_i = -Cv_i + U_i$. For an event-free interval $\Delta \geq 0$, define

$$E = e^{-C\Delta}, \quad A = (1 - E)/C, \quad B = (\Delta - A)/C. \quad (61)$$

Its exact real-arithmetic map is

$$v_i^+ = Ev_i + AU_i, \quad x_i^+ = x_i + Av_i + BU_i. \quad (62)$$

Integrating the velocity equation and then its solution proves this formula. Uniqueness of the linear ODE proves that splitting an interval with unchanged U_i yields the same endpoint. Numerically stable use of `expm1` can reduce cancellation; it does not change the exact identity.

Advance to an event before its ordered jump/update. A sample binds the actual state according to the declared pre/post ordering. Reception stores immutable values; commit uses only complete received frames. A priority jump changes h_i and its decoder together, preserving $c_i = x_i - m_i$. Cohort maps retain surviving x, v, m . An impulse adds its velocity jump without changing position. Retired/inactive bodies remain in state. Zero base communication latency uses the same generation/forward/reception semantics.

For a fixed finite admissible event trace and fixed numerical priority inputs, continuous and numerical snapshot/controller variables can be embedded into a finite vector z with affine event maps $z_{\ell+1} = T_\ell z_\ell + b_\ell$. A perturbed run on that same trace with local defects r_ℓ satisfies $e_{\ell+1} = T_\ell e_\ell + r_\ell$, hence

$$\begin{aligned} \|e_N\| &\leq \left(\prod_{\ell=0}^{N-1} \|T_\ell\| \right) \|e_0\| \\ &\quad + \sum_{j=0}^{N-1} \left(\prod_{\ell=j+1}^{N-1} \|T_\ell\| \right) \|r_j\|. \end{aligned} \quad (63)$$

Finite induction proves the bound; empty products are one. It needs actual certified local defects to certify a numerical output. It does not compare traces with different event selections or prove an infinite-horizon floating-point guarantee. Priority values were fixed inputs in this affine reduction: jointly perturbed conditioning maps are generally rational, not affine, and require their own error estimates. Pathwise admissibility of another trace does not provide a same-trace numerical error bound.

C. Conditional active-body tubes

In a final fixed-support phase after the coherent cut T_0 , put $P_{\text{all}} = \max(P, \sup_{[-H,0],i} \|\eta_i\|)$. For active balls of radius r_b , the condition

$$\min_{i \neq j} \|r_i - r_j\| > 2r_b + 2P_{\text{all}} \quad (64)$$

proves active disjointness. Indeed $x_i - x_j = r_i - r_j + \eta_i - \eta_j$ and the reverse triangle inequality bounds center distance strictly above $2r_b$. The nominal command has norm at most $K(P_{\text{all}} + Q) + CV_{\text{body}}$, since $\|m_i\| = \|\eta_i(\sigma) - \tilde{q}_i(\sigma)\| \leq P_{\text{all}} + Q$. An actuator limit at least this bound avoids saturation in that declared phase; physical acceleration also includes the disturbance. These are active-pair, final-phase conditions. They do not cover entry paths, retired-body collisions, obstacles, ground or rotor attitude.

Positive terminal spacing does not imply transient safety. Two agents with offsets $(-e, e)$, initial positions $(e, -e)$, equal goals, zero velocities and zero disturbances stay in the same symmetric line. Stable recovery makes η_1 go continuously from $2e$ to zero, so it passes through e when both positions coincide. The tube condition correctly excludes this swapped initial state.

D. The supplied rigid-body simulator

The supplied `HORTRAME_Flying_Cats_v1.5.1.html` has SHA-256

```
572650a79ebe84c26d409233c10b2186
b8513c7478de1fb199288da54e281aa0
```

Its inspected application path computes a centralized residual mean in `recomputeFormationOptimum`, reached through `updateFormationRecovery`'s velocity-direction trigger. It uses a global formation offset and a rigid-body `world.step` path. Source inspection is distinct from engine execution and a complete plant-error bound.

In a displaced zero-velocity fixture, radial trigger values are zero and `anyAway` is false. With recovery inactive, the actual extracted routine skips recomputation. Inward and outward fixtures discriminate the trigger. These are three actual function-callback fixtures, not complete browser/world trajectories. The code does not inherit the coherent distributed snapshot, counted clock/cohort services, retained-body semantics or point-plant safety theorem from resembling their variable names. The rigid-body bridge remains open until continuous and impulsive deviations are derived and bounded. Equation (62) is a mathematical nominal implementation contract; no new simulator mode has been implemented or validated by this article.

XII. VERIFICATION, DEPENDENCY STATE AND STOPPING RULE

The companion code uses deterministic mechanical checks. Its frozen records contain 54 exact rational fixtures for optimizer, translation, mean-error, cut, clock and shape identities; three actual extracted recovery-guard callback fixtures; and four finite nominal event simulations with uniform, directional unequal, state-dependent and zero-base state-dependent delays. The nominal cases retain actual generated immutable frames, duplicate/stale rejection, one admission and removal, physical state continuity and a continuing retired body. The balanced priority collection used in those fixtures is a special mean-preserving instance, not a universal mean claim for (45).

For the four nominal runs, $a = b = 0.2$, $h = 0.6$, $L = 0.1$, $H = 0.7$ and horizon 180 seconds. The maximum reported endpoint residual was $4.7574190133278185 \times 10^{-14}$ and maximum relative template residual was $2.5041559624044446 \times 10^{-14}$. They use floating-point evaluation of the nominal exact event formula, not a certified roundoff bound. Event-sampled separation and speed are not continuous-time safety proofs. No full-engine trajectory or hardware result is reported.

The proof dependencies are explicit: translation centering yields the modal equations; actual same-time complete frames and coherent commit yield zero mean estimation error; that invariant yields the delayed relative loop; the small-age bound and convergent shifted priority inputs yield recovery. Priority convergence separately depends on actual positive coefficients, used-age bounds and consumed joint connectivity. A physical rate additionally uses a lower action-count bridge. Final-cohort conclusions use finite correct record/bootstrap application and an actual retained-history cut. Local shape profiles alter the limiting relative input and are counted separately. Mean endpoint conditions, numerical certification and the rigid-body bridge are not consequences of priority agreement.

Adversarial review is conducted by the existing permanent TESSERA role on exact frozen objects. The first actual core review found an inconsistent impulse-cut convention, insufficient regular-forcing premises in an impulse extension and an incomplete infinite-time sampling partition. Those were corrected in the successor core and dependent modules; failed predecessors and counterexamples remain preserved. The actual full-stack R02 review found repeated-edge traffic and relative-impulse cut repairs. Full-source R03 cleared those affected repairs, retained the stated conditional mathematical scopes and required an explicit coherent analysis origin. This edition names T_0 , its actual retained history and clock phase, and the scored post-cut domain. Exact successor recheck dispositions are retained with their frozen objects. This manuscript additionally makes forwarding-opportunity counts and future impulse intervals explicit, narrows the affine numerical reduction to fixed priority inputs, and adds a template-invariance corollary. R03 reviewed the strong invariant-template corollary and the full mathematical source. R04 discharged the remaining analysis-cut and scope recheck at the exact successor identities; its accompanying disposition preserves the declared assumptions and open publication obligations. Shared task/model lineage and access to the same artifacts limit independence; agreement count is not independent proof replication.

The results are deductions under the stated assumptions; each adversarial disposition applies only to its exact frozen source and declared attack. No disposition asserts absence of every possible defect. Finite check results are observations at their recorded scopes. Literature/novelty, complete rigid-body realization, certified approximate arithmetic and unrestricted transient safety remain open or out of scope as stated. A stop requires completion of the declared applicable mathematical and manuscript attacks, repair and affected recheck of material findings, exact artifact freeze and required rendering verification. It does not require declaring all possible objections absent. Missing owner literature or human acceptance is

retained as an unresolved publication obligation, not silently promoted to mathematical closure.

XIII. CONCLUSION

Centering an instantaneous translation objective removes absolute positional stiffness from the common mode. Complete coherent residual snapshots and local innovation preserve this cancellation causally, while a sufficient age/gain condition stabilizes the relative sampled loop. Actual delayed priority agreement supplies convergent goal inputs without assuming a schedule-independent mean. Shared goals cancel from terminal shape; explicitly added local profiles provide a priority-dependent template. Finite support changes retain actual state and departed bodies, but guarantee recovery only after a final coherent history cut. Mean endpoints, active-body tubes, finite numerical error bounds and rigid-body realization have distinct hypotheses.

AI DISCLOSURE AND PENDING HUMAN ACCEPTANCE

The following owner-required paragraph is reserved for the human-accepted final manuscript. Its human-review statement is not a completed event in this preprint:

Generative artificial intelligence systems were used during development of the mathematical argument and manuscript. The system contributed to formulation and refinement of portions of the theorem, proof construction, adversarial examination of intermediate arguments, simulation-code development, and manuscript drafting. All mathematical claims, derivations, citations, simulation results, and final text were reviewed and accepted under the responsibility of the human author.

AI agents using the CALIBER and TESSERA assistant roles contributed to the mathematical modules and adversarial review; CALIBER assembled the manuscript and finite verification code. AI assistance affected the abstract, Sections I–XIII, this acknowledgment and accompanying verification code, including formulation, equations, proofs, counterexamples, scope analysis and nominal numerical reporting. These roles are not authors. The contribution description must be reconciled with the actual final manuscript and reviewed literature before human acceptance. No completed literature or human-review event is asserted here.