

A Sampled Common-Translation Latch for Nominal Double Integrators

Addendum to Translation-Equivariant Disturbance Recovery in Delayed Open Priority Networks

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Abstract—We study a sampled common-translation latch coupled to equal-gain nominal double integrators. The sampled branches follow the recovery callbacks in the Flying Cats version cited by the parent article: settled bodies are skipped, recomputation precedes a possible priority update, and the returning branch holds the translation while its timer advances. For a fixed finite cohort, convergent goals and vanishing disturbance integrals on fixed sampling cells, exact nominal flow and every sequence of holds or recomputations produce vanishing tracking error and velocity. The source-bound guard consequently stops recomputing after finitely many samples. The endpoint contains the actual frozen translation. We prove augmented translation equivariance, well-posedness, a last-recomputation error formula, and separate timer, impulse and bounded-residual statements. Complete-cohort information and a common clock are explicit resources. The result does not establish a rigid-body, airflow or floating-point realization.

Index Terms—formation control, sampled feedback, common translation, hybrid systems, disturbance recovery.

I. SCOPE AND MODEL

The parent [1] distinguishes an exact common-translation optimizer from a causal controller using coherent delayed snapshots. This addendum studies a common translation recomputed or retained at fixed samples. Retaining it introduces common restoring stiffness, so the endpoint theorem differs from the parent’s freely translating common-mode result.

The branches come from `formationProgress`, `recomputeFormationOptimum`, `updateFormationRecovery` and `physicsTick` in the parent-cited Flying Cats v1.5.1 [2]. Its program advances a rigid-body simulation. Here the branches are coupled to the exact nominal plant below. Branch correspondence and plant realization are separate claims.

After the last relevant cohort, slot and initialization change, fix $I = \{1, \dots, n\}$, $1 \leq n < \infty$, offsets $r_i \in \mathbb{R}^3$, and equal scalar gains $K, C > 0$. Parent gains $K = ab$, $C = a + b$, $a, b > 0$, are included. The clock is $t_k = t_0 + kh$, $h > 0$, with unbounded physical analysis time. Between latch operations,

$$\dot{x}_i = v_i, \quad \dot{v}_i = K(r_i + q_i(t) + \hat{s}(t) - x_i) - C v_i + d_i(t). \quad (1)$$

Goals are prescribed locally bounded measurable signals with finite one-sided sample values. Ordinary disturbances are locally integrable. Only the common latch $\hat{s} \in \mathbb{R}^3$ is held between samples; goals may vary inside a cell. Exogenous discrete events are locally finite or represented by the admitted signals. No full initialization reset occurs after the cut.

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Bars denote unweighted cohort means. Put

$$\begin{aligned} p_i &= x_i - r_i, & \xi &= \bar{p}, & \eta_i &= p_i - \xi, \\ s^* &= \xi - \bar{q}, & c_i &= r_i + q_i + \hat{s}. \end{aligned} \quad (2)$$

Thus the physical centre is $\bar{x} = \bar{r} + \xi$. At a sample the order is plant flow and any endpoint velocity impulse, guard/reset using q_k^- , then any priority update to q_k^+ . Position and velocity are unchanged by guard/reset and priority operations. Superscripts identify stages within one physical time.

Successful recomputation uses the same-stage x_i, r_i, q_i^- of the complete ordered cohort. Complete-cohort sensing or a correct common service, shared spatial conventions and the clock are counted resources. This construction supplies no distributed discovery protocol and does not inherit the parent’s different causal decoder or its communication contracts.

II. MODAL IDENTITIES AND EQUIVARIANCE

Lemma A1 (Common-offset cancellation). *For every admissible common latch, almost everywhere between operations,*

$$\ddot{\xi} + C\dot{\xi} = K(\hat{s} - s^*) + \bar{d}, \quad (3)$$

$$\ddot{\eta}_i + C\dot{\eta}_i + K\eta_i = K(q_i - \bar{q}) + (d_i - \bar{d}). \quad (4)$$

Proof. Average (1), using $\bar{r} + \bar{q} + \hat{s} - \bar{x} = \bar{q} + \hat{s} - \xi$. This gives (3). Subtract that average from each equation for p_i ; equal gains and the identical latch cancel the common term. Latch operations change neither position nor velocity, so the relative state has no reset jump. $\square$

Unequal gains, different applied latches, saturation or another plant require a residual or separate bridge. This is the direct nominal modal identity, not a realization of the parent’s local innovation controller.

Proposition A2 (Augmented translation equivariance). *For fixed $\Delta \in \mathbb{R}^3$, transform $(x_i, \hat{s}) \mapsto (x_i + \Delta, \hat{s} + \Delta)$, retaining velocities, offsets, prescribed goals/disturbances, clock, availability, flag and timer. The nominal flow and the sampled law below give translated trajectories with identical flag/timer history.*

Proof. Both c_i and x_i increase by Δ , leaving error and all guards unchanged. The optimum s^* increases by Δ , so recomputation respects the transform; holds do also. Acceleration in (1) is unchanged. Uniqueness of each linear flow and determinism of successive branches propagate the relation. $\square$

Displacement with a fixed latch is different. No equivariance of an untranslated floor, obstacle, radio/airflow environment, full reset routine or numerical rounding follows.

III. THE SOURCE-BOUND SAMPLED LAW

For the cited instance use, in metres and seconds as appropriate,

$$\begin{aligned} R &= .55, & V &= .22, & \alpha &= .12, & \beta &= .05, \\ \varepsilon &= 10^{-9}, & \tau &= .24, & h &= 1/120. \end{aligned} \quad (5)$$

The convergence proof also allows fixed $h > 0$ and positive R, V, α, τ . State is the latch, flag $b \in \{0, 1\}$ and finite timer $H \geq 0$. Let E_k be supplied availability. At the pre-priority guard define

$$e_i^k = r_i + q_i^{k,-} + \hat{s}^{k,-} - x_i^k, \quad S_i^k = [\|e_i^k\| \leq R \wedge \|v_i^k\| \leq V]. \quad (6)$$

For unsettled bodies,

$$\nu_i^k = \begin{cases} (v_i^k)^\top e_i^k / \|e_i^k\|, & \|e_i^k\| > \varepsilon, \\ 0, & \|e_i^k\| \leq \varepsilon. \end{cases} \quad (7)$$

Set

$$A_k = \bigvee_{i: \neg S_i^k} [\nu_i^k < -\alpha], \quad B_k = \bigwedge_{i: \neg S_i^k} [\nu_i^k > \beta]. \quad (8)$$

Empty reductions give $A_k = 0$, $B_k = 1$. Settled bodies are skipped.

Proposition A3 (Recovery branches and caller order). *The real-arithmetic transcription of the cited recovery callbacks is:*

- 1) $E_k = 0$: retain latch, flag and timer.
- 2) $E_k = 1, b^- = 0, A_k = 0$: retain all three.
- 3) $E_k = 1, b^- = 0, A_k = 1$: set $b^+ = 1, H^+ = 0$ and $\hat{s}^+ = \xi^k - \bar{q}^{k,-}$.
- 4) $E_k = 1, b^- = 1, B_k = 0$: set $H^+ = 0, \hat{s}^+ = \xi^k - \bar{q}^{k,-}$ and retain $b^+ = 1$.
- 5) $E_k = 1, b^- = 1, B_k = 1$: retain the latch and increment H by h . If $H \geq \tau$, set $b^+ = 0, H^+ = 0$; otherwise retain $b^+ = 1$.

The source bypasses in trial mode, before grace or with empty cohort. Initial recovery reset sets $\hat{s} = 0, b = 0, H = 0$ and 0.75-second grace. Successful recomputation requires matching nonempty goal/cohort metadata, as assumed in the final-cohort model.

Proof. The source skips settled bodies, uses the radial cutoff and comparisons in (8), resets the timer in both recomputing branches, and holds the offset in the returning branch. The recomputation assigns the mean of $x_i - r_i - q_i$. Its caller executes the plant step, recovery, and priority consumption/target refresh in that order. These assignments give the stated branches and pre/post-priority stages. $\square$

The source uses JavaScript floating-point arithmetic and a rigid-body step. This proposition identifies its logic in ideal arithmetic; threshold roundoff, sensing and nominal tracking remain separate. An active flag can already hold the latch. Availability may be arbitrary; enough enabled calls are required only for the separate flag-exit conclusion.

Proposition A4 (Supplied-input well-posedness and no Zeno switching). *For the prescribed admissible signals, availability and locally finite event order in Section I, each finite*

physical/latch state and finite nonnegative timer gives one global nominal trajectory. Every $[t_0, T]$ contains at most $\lfloor (T - t_0)/h \rfloor + 1$ sampled guard opportunities.

Proof. With constant latch on a cell, the plant is linear with locally integrable forcing. Variation of constants gives one finite solution. Each sampled branch has deterministic comparisons, the zero-error convention and finite affine assignments. Induction over the finitely many samples on each compact interval gives the trajectory and count. Admitted locally finite exogenous events preserve the argument. $\square$

The positive fixed grid supplies the no-Zeno property, not hysteresis alone or an event index accumulating in finite physical time. Well-posedness of an unspecified state-dependent priority, sensing or airflow service is not asserted.

IV. LATCH AGE AND FROZEN ENDPOINTS

Lemma A5 (True-last-recomputation error). *If the last successful recomputation was at stage u , with $\hat{s} = \xi(u) - \bar{q}(u^-)$, and none has occurred later, then*

$$\hat{s} - s^*(t) = -[\xi(t) - \xi(u)] + [\bar{q}(t) - \bar{q}(u^-)], \quad (9)$$

$$\|\hat{s} - s^*(t)\| \leq \int_u^t \|\bar{v}(s)\| \, ds + \|\bar{q}(t) - \bar{q}(u^-)\|. \quad (10)$$

For piecewise locally absolutely continuous goals with locally finite jumps,

$$\|\hat{s} - s^*(t)\| \leq \int_u^t \|\bar{v} - \dot{\bar{q}}_{ac}\| \, ds + \sum_{u \preceq s \preceq t} \|\Delta \bar{q}(s)\|. \quad (11)$$

The stage order includes a priority jump following the reset at u .

Proof. Subtract $s^*(t)$ from the retained reset and integrate $\xi' = \bar{v}$. The triangle inequality gives (10); the absolutely continuous and jump decomposition gives (11). No derivative of a merely measurable goal is used. $\square$

An age- h derivative bound applies only between adjacent recomputation samples. Consecutive enabled returning calls complete the exact source timer after $\lceil \tau/h \rceil = 29$ increments. Bypassed calls retain the timer while time advances, so $29h$ is not a universal active-age bound. A frozen latch has unbounded age.

Theorem A6 (Frozen-mode endpoint). *If $\hat{s} = \hat{s}_\infty$ after finite time, $q_i(t) \rightarrow q_{i\infty}$ and $d_i(t) \rightarrow 0$, then*

$$x_i(t) \rightarrow r_i + q_{i\infty} + \hat{s}_\infty, \quad v_i(t) \rightarrow 0, \quad (12)$$

and $\xi(t) \rightarrow \bar{q}_\infty + \hat{s}_\infty$.

Proof. Subtract the constant target. The resulting oscillator has matrix $\begin{bmatrix} -\frac{0}{KI} & -\frac{I}{CI} \end{bmatrix}$, with characteristic roots of negative real part for $K, C > 0$. Its forcing $K(q_i - q_{i\infty}) + d_i$ tends to zero. In the exponentially weighted convolution, the finite early part vanishes and the tail is bounded by an arbitrarily small input norm times an integrable kernel. Initial response also vanishes. $\square$

Integrability of the whole mean-force tail is unnecessary here; the parent's free common mode has another endpoint

condition. If $d_i \rightarrow D_i$ instead, the same calculation adds position shift D_i/K , not zero error.

V. EVENTUAL ABSENCE OF RECOMPUTATION

Stack $Q = (q_1, \dots, q_n)$ and $D = (d_1, \dots, d_n)$ in $\mathbb{R}^{3n}$.

Theorem A7 (Convergence and eventual latch freeze). *Assume the fixed finite nonempty cohort, aligned metadata, exact nominal flow and unbounded fixed sampling grid above. Availability may be arbitrary. If $q_i(t) \rightarrow q_{i\infty}$ and*

$$\int_{t_k}^{t_{k+1}} \|D(s)\| \, ds \rightarrow 0, \quad (13)$$

there is a finite sample after which no recomputation occurs. The actual latch is then a finite constant $\hat{s}_\infty$, and (12) holds. An active flag exits if enough subsequent enabled calls complete its remaining timer.

Proof. Define the analysis-only state

$$\begin{aligned} z_i &= x_i - r_i - q_{i\infty} - \hat{s}, & Z_b &= (z_1, \dots, z_n), \\ V_b &= (v_1, \dots, v_n), & Y &= (Z_b, V_b), \\ \|Y\|_P^2 &= K \|Z_b\|^2 + \|V_b\|^2. \end{aligned} \quad (14)$$

The controller need not know $q_{i\infty}$. Between resets,

$$\begin{aligned} \dot{Y} &= \mathcal{A}Y + \mathcal{B}F, & F &= K(Q - Q_\infty) + D, \\ \mathcal{A} &= \begin{bmatrix} 0 & I \\ -KI & -CI \end{bmatrix}, & \mathcal{B} &= \begin{pmatrix} 0 \\ I \end{pmatrix}. \end{aligned} \quad (15)$$

Homogeneous flow satisfies

$$\frac{d}{dt} \|Y\|_P^2 = -2C \|V_b\|^2. \quad (16)$$

Every nonzero state loses energy over any fixed positive interval: equality would force continuous $V_b = 0$ throughout, then $\dot{Z}_b = 0$ and $\dot{V}_b = -KZ_b = 0$, contradicting a nonzero initial state. Compactness of the P -unit sphere and flow continuity imply

$$\sigma_h = \|e^{\mathcal{A}h}\|_P < 1, \quad \|e^{\mathcal{A}s}\|_P \leq 1 \quad (s \geq 0). \quad (17)$$

Since $\|\mathcal{B}F\|_P = \|F\|$, variation of constants gives

$$\begin{aligned} \|Y(t_{k+1}^-)\|_P &\leq \sigma_h \|Y(t_k^+)\|_P + U_k, \\ U_k &= \int_{t_k}^{t_{k+1}} \|F(s)\| \, ds \rightarrow 0. \end{aligned} \quad (18)$$

Goal convergence and (13) justify the limit.

Let $u = (1, \dots, 1)^\top \in \mathbb{R}^n$ and $\Pi = (I_n - uu^\top/n) \otimes I_3$. Recomputation preserves velocity and maps

$$Z_b^+ = \Pi Z_b^- + u \otimes (\bar{q}_k^- - \bar{q}_\infty). \quad (19)$$

Indeed $\xi = \bar{q}_\infty + \hat{s}^- + \bar{z}^-$; substitute the new latch $\xi - \bar{q}_k^-$ into (14). Centering positions is a P -orthogonal, nonexpansive projection. A hold is the identity. Thus $r_k = \|Y(t_k^+)\|_P$ obeys, for every reset or hold,

$$r_{k+1} \leq \sigma_h r_k + U_k + J_{k+1}, \quad J_k = \sqrt{nK} \|\bar{q}_k^- - \bar{q}_\infty\| \rightarrow 0. \quad (20)$$

For $\delta > 0$ choose k_1 so $U_k + J_{k+1} \leq \delta$ thereafter. Iteration gives

$$r_k \leq \sigma_h^{k-k_1} r_{k_1} + \delta \sum_{j=0}^{k-k_1-1} \sigma_h^j.$$

Hence $\limsup r_k \leq \delta/(1 - \sigma_h)$; let $\delta \downarrow 0$. Inside a cell, $\|Y(t)\|_P \leq r_k + U_k \rightarrow 0$. Convergence has been proved for arbitrary holds and resets, without assuming freeze.

At every sufficiently late guard, $e_i^k = q_i^{k,-} - q_{i\infty} - z_i^{k,-}$ and v_i^k have norms below R and $\min(V, \alpha)$. Every body is settled, so $A_k = 0, B_k = 1$. Both enabled flag states and every bypass now hold the latch. No future recomputation occurs. There were finitely many earlier samples, all finite, so the retained latch is finite. The already-established $Y(t) \rightarrow 0$ directly yields (12); Theorem A6's stronger pointwise premise is not invoked.

If active at this cut with timer H , at least $\max\{1, \lceil(\tau - H)/h\rceil\}$ later enabled calls suffice to exit. Source-reachable inactive states have $H = 0$, active states $0 \leq H < \tau$. The maximum with one also covers arbitrary finite nonnegative initial timers. No elapsed-time exit bound follows without an availability contract. $\square$

For constant goals and zero force from a cut, $U_k = J_k = 0$, $r_k \leq \sigma_h^k r_0$. A sufficient no-recomputation sample satisfies

$$\sigma_h^k r_0 < \min\{\sqrt{K}R, V, \alpha\}. \quad (21)$$

This may be conservative; merely convergent inputs give no uniform settling time. Cell-integral vanishing is weaker than pointwise vanishing: on cell k a scalar pulse of height k and width h/k^3 has integral $h/k^2 \rightarrow 0$, despite growing peaks.

Corollary 1 (Integral forcing and velocity impulses). *Replace $D(t) \, dt$ by a locally finite vector measure M . Use continuous positions and right-continuous locally BV velocities satisfying the integral plant equation. Cut at actual post-atom data. Future cells are $(t_k, t_{k+1}]$, with endpoint atoms before the guard. If $|M|((t_k, t_{k+1}]) \rightarrow 0$ and goals converge, Theorem A7 holds. Finite future total variation, including finitely many finite velocity impulses, is sufficient.*

Proof. Variation of constants adds $\int_{(t_k, t_{k+1}]} e^{\mathcal{A}(t_{k+1}-s)} \mathcal{B} M(ds)$. Its P -norm and each partial-cell term are bounded by cell total variation. Replace U_k by the goal-error integral plus that variation. The same projection/recursion proof applies. Disjoint cell variations of a finite future measure tend to zero. $\square$

This is an integral-solution result, not a classical derivative across atoms or a contact realization.

Corollary 2 (Conditional nominal residual margin). *If the same flow/reset model has $\limsup U_k \leq U$, $\limsup J_k \leq J$, and whole-cell forcing integral asymptotically bounded by U , then*

$$\begin{aligned} \limsup r_k &\leq \frac{U + J}{1 - \sigma_h}, \\ \limsup_{t \rightarrow \infty} \|Y(t)\|_P &\leq B_* := \frac{U + J}{1 - \sigma_h} + U. \end{aligned} \quad (22)$$

If $q_i \rightarrow q_{i\infty}$ and $B_* < \min\{\sqrt{K}R, V, \alpha\}$, no sufficiently late recomputation occurs. Flag exit requires enabled calls. A positive tube bound does not imply zero tracking error.

Proof. Iterate (20) from a late cut with forcing bound $U + J + \delta$, let initial response decay, then let $\delta \downarrow 0$. The partial-cell estimate adds at most U . Since $\|z_i\| \leq \|Y\|_P / \sqrt{K}$ and $\|v_i\| \leq \|Y\|_P$, the strict margin and goal convergence place all late guards in the settled region. $\square$

The corollary assumes a justified nominal acceleration residual. It does not prove its existence, size or decay for a rigid body, actuator, airflow/contact solver or numerical integrator.

VI. REACTIVATION AND PRIORITY ENDPOINTS

Proposition A8 (Reactivation and a final admissible quiet tail). *At an enabled sample with inactive flag, reactivation/recomputation occurs iff an unsettled body has $v_i < -\alpha$. Finitely many finite disturbance episodes followed by Theorem A7's tail assumptions give eventual recovery/freeze. Displacement magnitude alone need not reactivate.*

Proof. The first claim is branch 3. Every admissible finite prefix has finite state by Proposition A4; apply Theorem A7 on the final tail. For the last claim take $n = 1$, $r = q = \hat{s} = 0$, inactive flag, zero disturbance and $K = ab$, $C = a + b$. Let $x(0) = De$, $v(0) = 0$, with $D > 0$ and unit e . If $a \neq b$,

$$x(t) = D \frac{be^{-at} - ae^{-bt}}{b - a} e;$$

if $a = b$, $x(t) = D(1 + at)e^{-at}e$. The scalar coefficient is nonnegative and nonincreasing. Velocity is inward, so $\nu = -\dot{x}_{\text{scalar}} \geq 0$ where the radial formula applies, and zero otherwise. No outward guard occurs for arbitrarily large D . The body recovers to the old target without recomputation. $\square$

Episodes may merge if quiet gaps are short. Common resets cancel in (4), but changed motion may change a motion-dependent priority schedule.

For an actual admissible delayed priority execution $w_i \rightarrow \mu_\sigma$, shared anchors give $q_{i\infty} = \mu_\sigma Z$ and

$$\bar{x}_\infty = \bar{r} + \hat{s}_\infty + \mu_\sigma Z. \quad (23)$$

With separately supplied fixed local profiles Z_i ,

$$x_{i\infty} - x_{j\infty} = r_i - r_j + \mu_\sigma (Z_i - Z_j). \quad (24)$$

These are pathwise applications of the parent's priority convergence hypotheses. μ_σ is distinct from the physical translation $\hat{s}_\infty$. Absence of a direct disturbance term in the priority law does not imply a common endpoint across motion-dependent schedules.

VII. SOURCE CHECKS, LIMITS AND CONCLUSION

The supporting script extracts six functions from the exact HTML and runs 14 finite callback fixtures with declared vector/body/service stubs. Cases cover settled skipping, strict thresholds, cutoff, cohort means, 29 returning calls, bypass age, large initial timers, empty/mismatched metadata, actual caller order, post-reset goal jumps, augmented translation and

displacement at rest. One initial activation fixture did not satisfy its own outward threshold; its velocity was corrected, preserving the failed result. The checks concern those branches, not the complete engine or universal numerical correctness.

The analysis uses exact continuous flow and an exact common latch. Attitude, thrust/tilt/speed limits, avoidance, contacts, obstacles, sensing and floating-point thresholds need separate realization proofs. A zero wind command alone does not establish airflow, rotor, turbulence or contact-force decay. No collision, hardware, minimum-resource or novelty claim is made.

For this nominal model, flow contracts the energy norm over each fixed cell and recomputation is a nonexpansive position projection plus a vanishing goal-limit term. Convergence follows before any freezing assumption. The actual settled-body guard then stops recomputing, yielding the endpoint with retained translation. Flag exit and physical realization retain their separate conditions.

AI-USE DISCLOSURE

This work was developed with assistance from AI agents operating within the HORTAME Systems research architecture. Depending on the research stage, these agents contributed to literature analysis, mathematical formulation, proof reconstruction, adversarial review, counterexample search, simulation and code analysis, verification design, reproducibility checks, revision, and manuscript preparation. AI-agent outputs were treated as research artifacts subject to verification and correction rather than as independent authority. G. Bolduc is the human author responsible for the manuscript, its claims, source selection, and publication decisions. AI-agent review does not constitute independent human peer review, proof-assistant certification, experimental validation, or publication acceptance.

CALIBER and TESSERA contributed proof/source analysis, counterexamples, adversarial repair, finite callback checks and manuscript preparation. They share the parent/source frame and exchanged the energy argument; agreement is not independent human review.

REFERENCES

- [1] G. Bolduc, *Translation-Equivariant Disturbance Recovery in Delayed Open Priority Networks*, HORTAME Systems, supplied preprint, ver. 05, Oct. 2026. Parent source SHA-256 prefix 165c9b5d; full identity in the supporting source manifest.
- [2] HORTAME, *Flying Cats*, supplied HTML, ver. 1.5.1. Parent-cited source SHA-256 prefix 572650a7; recovery functions/caller in the supporting source contract.