

Common-Mode Shielding During Finite Population Changes

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Abstract—We study a nominal admission controller with a nonempty protected cohort and finitely many shielded newcomers. Complete coherent residual tables centered on the protected cohort cancel all shielded physical residuals from its mean acceleration. Protected departures and gated promotions have explicit position and velocity coordinate maps. Each population or role event invalidates the preceding residual epoch and uses a bounded zero-correction waiting law. With finitely many born incarnations, at most one promotion per incarnation, the stated delayed-priority contracts, and the parent factored-gain small-age condition, a final continued shield phase captures every remaining newcomer. A finite group release is then admissible, and final relative geometry recovers with position and velocity retained at the switch. Uniform protected-relative and event-seam bounds require an explicit family of uniformly bounded histories, service, forcing, event counts and stability margins; capture time and newcomer effort may grow with initial distance. We derive the final group-release mean response and distinguish summable limiting shifts from summable whole-response envelopes. Proofs concern exact nominal dynamics. New finite rational checks corroborate selected algebra only; no online release detector, collision, actuator, radio, rigid-body or perpetual-churn result is established.

Proposed IEEE reference: G. Bolduc, “Common-Mode Shielding During Finite Population Changes,” *HORTRAME Research*, HORTRAME Systems, internal preprint, ver. 0.3, Oct. 2026. Public locator and human acceptance are pending; no DOI is asserted.

Index Terms—formation control, delayed feedback, population changes, staged admission.

I. SOURCE FRAME AND MODEL

The starting point is the single-newcomer shield [1]. Its delayed relative argument comes from the complete coherent snapshot construction and E1–E2 recovery theorem in [2]. The priority input uses the actual causal delayed process of [3], rather than interpreting the synchronous fixed-epoch matrix of [4] as an asynchronous algorithm. Source identities are bound in the review package to the seven originals in the owner-supplied archive. In particular, the supplied [2] is the cb0d7247... revision; older parent hashes quoted inside the addenda remain historical citations, not an assertion of byte equality.

Let a finite set of physical bodies have fixed offsets $r_k \in \mathbb{R}^3$. Their active incarnations form $A = I \dot{\cup} J$, where $I \neq \emptyset$ is protected and J is shielded. Each body has at most one active incarnation. An incarnation enters J once, may promote to I at most once, and may leave; no demotion or incarnation reuse occurs inside an episode. The initial active set and number

of arrivals are finite. A new incarnation can be assigned to an existing finite-state body after its preceding incarnation’s terminal retirement; its identity is fresh and its law changes without a plant reset. A retired body remains in physical state under the declared separate law $\dot{x} = v$, $\dot{v} = -Cv + d_{\text{ret}}$, stops active priority/control operations, and rejects later packets addressed to the retired incarnation. This law is no collision or parking guarantee.

Every surviving body retains position and velocity at events. New physical states are finite, not reset to the incumbent center. Offsets and gains do not change during the episode. Events have a locally finite total order, including an internal order for equal physical timestamps; a finite batch may be one explicitly defined combined departure/promotion event. A correct, sufficiently preannounced record lets every active controller apply the new physical partition and waiting law coherently at the declared activation time. This is a counted timing/cohort service, not instantaneous detection of an unannounced failure. The resulting protected set must be nonempty. Newly arriving incarnations cannot be directly promoted.

The priority process has ordered locally applied objective supports, positive normalized rows when initialized, and fixed per-objective anchor vectors. Z_{S_k} lists those anchors in participant k ’s applied support order and $q_k = w_k Z_{S_k}$. Support maps and transported priority history obey [3], [4]. A newborn without coherent positive bootstrap has no ready priority row and remains under the waiting law; it is not assigned a fictitious row. The first complete new-epoch residual table additionally requires coherent positive goal/support bootstrap for every active source within the service budget. Promotions alone change shield roles, not objective semantics. Anchors are bounded independently of newcomer starting positions; the diameter of their common hull supplies a bound Q on $\|q_k - \bar{q}_I\|$ wherever the initialized goals are used. Bodies and objectives need not have the same count in a fixed-support calculation.

Write

$$p_k = x_k - r_k, \quad h_k = p_k - q_k, \quad \bar{y}_B = |B|^{-1} \sum_{i \in B} y_i, \quad (1)$$

$$\xi_I = \bar{p}_I, \quad \eta_i = p_i - \xi_I \quad (i \in I), \quad e_j = p_j - \xi_I \quad (j \in J), \quad (2)$$

$$\tilde{q}_i = q_i - \bar{q}_I, \quad g_j = q_j - \bar{q}_I. \quad (3)$$

Means, relative coordinates and goal metadata belong to the stated shield epoch. Ordinary disturbances are locally integrable; goals are locally bounded measurable signals. Positions are continuously differentiable and velocities locally absolutely

continuous between ordinary membership/role events, with equations holding almost everywhere. No velocity impulse branch is used here.

A. Exact gain and priority contracts

Retain the parent gains

$$K = ab, \quad C = a + b, \quad a, b > 0. \quad (4)$$

The position impulse kernel of the stable undelayed relative operator is

$$g(t) = \begin{cases} (e^{-at} - e^{-bt})/(b - a), & a \neq b, \\ te^{-at}, & a = b, \end{cases} \quad t \geq 0. \quad (5)$$

It has $\|g\|_{L^1} = 1/K$ and

$$G_v = \|g'\|_{L^1} = 2 \max_{t \geq 0} g(t), \quad \kappa = KHG_v < 1. \quad (6)$$

For $a = b$, $G_v = 2/(ae)$. Merely $K, C > 0$ does not imply this factored-gain proof domain: $K = C = 1$ has $C^2 - 4K = -3$. A general underdamped delayed theorem is not claimed.

On a final objective/physical support, the actual priority law has stochastic coefficient rows, current self-weights at least $\zeta > 0$, all positive used off-diagonal weights at least ζ , used ages at most D_p , and strongly connected unions of actually consumed influence in every B_p action indices. Initial augmented histories are actual positive final-support rows, and packets used by the recurrence must exist. The locally finite embedding of infinitely many actions in physical time tends to infinity, so $k(t) \rightarrow \infty$. The common-column argument of [3, Theorem 2], [2, P1] gives a common row μ and hence $\tilde{q}_i, g_j \rightarrow 0$ for shared anchors. No arithmetic-mean or schedule-independent endpoint is inferred. A physical exponential rate would additionally need a lower action-density bound, which this article does not claim.

II. COHERENT SERVICE AND SHIELD LAW

Each complete origin table binds the shield epoch, protected and active ordered sets, priority support/goal version, source time, and every source incarnation. Its entries are the actual same-time $h_k(\sigma)$. At use, every active controller uses the same complete committed table, never a mixture of epochs or incomplete origins. Reception may be asynchronous; application is coherent. Actual sampling/commits are locally finite and cover the future after bootstrap, with

$$0 \leq t - \sigma(t) < H. \quad (7)$$

The parent shared sample/commit clock is a sufficient realization. Storage is a complete origin table plus a pending table and metadata, $O(|A|)$ vectors per participant and $O(|A|^2)$ across the cohort. Queue load, in-flight copies, priority traffic, growing counters and exact-coordinate representation remain additional resources [2]. No minimum-resource or arbitrary-radio realization is asserted.

Define the correction

$$\chi_k(\sigma) = h_k(\sigma) - \bar{h}_I(\sigma), \quad k \in A. \quad (8)$$

During coherent capture use

$$\dot{x}_k = v_k, \quad \dot{v}_k = -K\chi_k(\sigma(t)) - Cv_k + d_k(t). \quad (9)$$

If a service provides full-A centered values $m_k = h_k - \bar{h}_A$, then $\chi_k = m_k - \bar{m}_I$ exactly. This identity still requires the complete coherent origins and known protected identities; it supplies no uncounted centroid oracle.

Lemma 1. Modal cancellation

For every eligible table, $\sum_{i \in I} \chi_i = 0$, $\chi_i = \eta_i - \tilde{q}_i$, and $\chi_j = e_j - g_j$. Consequently

$$\ddot{\xi}_I + C\dot{\xi}_I = \bar{d}_I, \quad (10)$$

$$\ddot{\eta}_i + C\dot{\eta}_i = -K\eta_i(\sigma) + K\tilde{q}_i(\sigma) + d_i - \bar{d}_I, \quad (11)$$

$$\ddot{e}_j + C\dot{e}_j = -Ke_j(\sigma) + Kg_j(\sigma) + d_j - \bar{d}_I. \quad (12)$$

Proof:

Subtracting the protected residual mean gives the two coordinate identities. Its protected sum is zero. Average the protected body equations, then subtract their mean from each active equation. Equal gains and coherent use are essential. No limit argument is used. ■

The unpromoted bodies' absolute residuals have no direct term in the protected mean acceleration. They can still affect priority values or a motion-dependent schedule; those indirect effects are bounded or excluded only by the relevant stated contract.

Theorem 1. Capture on a continued fixed shield epoch

Suppose a shield epoch is continued indefinitely with finite $I \neq \emptyset, J$, the service and gain contracts above, finite actual histories, final-support priority convergence, and bounded differential disturbances $d_k - \bar{d}_I$ tending to zero for every $k \in A$. Then $\eta_i, \dot{\eta}_i \rightarrow 0$ and $e_j, \dot{e}_j \rightarrow 0$. This is a continued-shield statement; it does not describe shield coordinates after an actual release terminates that law.

Proof:

Choose an actual analysis cut T_0 after bootstrap and the priority final-support cut, late enough that the whole $[T_0 - H, T_0]$ controller history and every used source have the stated fixed epoch/support. Rebase time for analysis; retain actual states, eligible stored frames and inherited clock phase. Nothing is reset.

Each coordinate $y = \eta_i$ or e_j obeys $\ddot{y} + C\dot{y} = -Ky(\sigma) + f$, where $f = Kq_{\text{rel}}(\sigma) + d_{\text{rel}}$ is bounded and tends to zero. Rewrite the stable undelayed operator as

$$\ddot{y} + C\dot{y} + Ky = K[y(t) - y(\sigma(t))] + f(t). \quad (13)$$

If V_- bounds actual history velocity and H_v, H_x are the suprema of the homogeneous velocity/position responses from actual cut data, then on every finite horizon

$$\begin{aligned} \sup_{t \geq -H} \|\dot{y}(t)\| &\leq V := \frac{V_- + H_v + G_v \|f\|_\infty}{1 - \kappa}, \\ \sup_{t \geq 0} \|y(t)\| &\leq H_x + HV + \|f\|_\infty / K. \end{aligned} \quad (14)$$

Indeed $\|y(t) - y(\sigma)\| \leq H \sup \|\dot{y}\|$; apply variation of constants with g' , combine history/current suprema, and solve the small-gain inequality. Position follows using g . The bounds do not depend on the horizon. Let $V_\infty = \limsup \|\dot{y}\|$. Splitting convolutions into a finite prefix and a late tail yields $V_\infty \leq \kappa(V_\infty + \delta)$ for every $\delta > 0$, because $\sigma(t) \geq t - H$ and the exogenous input vanishes. Hence velocity and delay error vanish; the position convolution then tends to zero. Priority convergence supplies the vanishing goal input.

An incumbent's bound uses its protected-relative history, not a shielded newcomer's capture history. A newcomer's own bound, effort and capture time may depend on its finite initial displacement. A finite common position endpoint requires a separate mean-forcing integrability condition; vanishing differential forcing alone does not supply it.

III. EVENTS, COORDINATE SEAMS AND CAUSAL CUTS

At time T , let $L = I \setminus I'$ be departing protected incarnations and $P = I' \setminus I \subseteq J$ promoted pre-event incarnations. Then $I' = (I \setminus L) \cup P \neq \emptyset$. Births enter J' , and shielded departures change only J , unless combined with another declared event.

Lemma 2. Exact protected-frame map

At the event, with no position/velocity reset,

$$\begin{aligned} \Delta\xi &= \xi_{I'} - \xi_I = \frac{\sum_{j \in P} e_j - \sum_{\ell \in L} \eta_\ell}{|I'|}, \\ \Delta v &= \bar{v}_{I'} - \bar{v}_I = \frac{\sum_{j \in P} \dot{e}_j - \sum_{\ell \in L} \dot{\eta}_\ell}{|I'|}. \end{aligned} \quad (15)$$

Surviving protected, remaining shielded and promoted coordinates respectively subtract $\Delta\xi$ and Δv from their old relative position and velocity. Promoted coordinates change from e_j to $\eta'_j = e_j - \Delta\xi$.

Proof:

In the new protected sum retain $n\xi_I - \sum_{\ell \in L} p_\ell + \sum_{j \in P} p_j$, where $n = |I|$. Substitute $p_\ell = \xi_I + \eta_\ell$, $p_j = \xi_I + e_j$, and $n - |I'| = |L| - |P|$. Common terms cancel. The same calculation on velocities proves the second identity. Coordinate subtraction gives the remaining statements.

Thus

$$\begin{aligned} \|\Delta\xi\| &\leq \frac{\sum_{j \in P} \|e_j\| + \sum_{\ell \in L} \|\eta_\ell\|}{|I'|}, \\ \|\Delta v\| &\leq \frac{\sum_{j \in P} \|\dot{e}_j\| + \sum_{\ell \in L} \|\dot{\eta}_\ell\|}{|I'|}. \end{aligned} \quad (16)$$

These algebraic bounds contain no direct unpromoted-newcomer term. Uniform independence from original distances additionally requires Section 5's family. Adding/removing only shielded bodies has $\Delta\xi = \Delta v = 0$. Ungated promotion with $e_j = Ru$ creates seam $Ru/(n+1)$, so arbitrary immediate promotion has no uniform distance bound.

An old centered table is not automatically valid after departure: old corrections $(a, -a)$ sum to zero on two incumbents, but retaining the first correction after the second departs gives nonzero protected mean a . New centering of the singleton gives zero. This is a failure of cohort reuse, not a physical position jump.

A. New epoch and waiting law

Advance the physical plant to the ordered event, check the promotion premise, apply the declared cohort/role change, increment the shield epoch, and invalidate every preceding committed correction and pending commit. Clear their eligibility, not their historical evidence. Preserve physical state and priority state under its own protocol. Recompute neither a mixed-epoch table nor an old correction implicitly.

Use $\dot{x}_k = v_k$, $\dot{v}_k = -Cv_k + d_k$ for active bodies until the first complete post-event table. Its actual source time must be at or after T , under the new epoch, partition and goal metadata. Later sources satisfy the same eligibility and age rules; older tables cannot replace a newer committed table. Wrong incarnations/sets/epochs, incomplete tables and inconsistent source metadata are rejected; identical duplicates add no second origin term. Priority prefixes and shield epochs are distinct identifiers.

The nominal service also applies the age bound to pending-table eligibility and release certification. A relevant pending table is one still eligible to become the committed correction or enter a release certificate in the live epoch. At time t , its actual source time $s \leq t$ must satisfy $t - s < H$. At age H it becomes permanently ineligible for both purposes, even if its historical bytes are retained. A newer committed source permanently supersedes every older pending source for physical use and release certification. Supersession or expiry never changes physical state. These are explicit controller/service rules, not consequences of local finiteness or bounded age of already used tables alone. The parent shared-clock realization satisfies them: its sole pending epoch commits at latency $L < H$, and prior sources cannot replace it. No additional pending-resolution deadline is assumed.

Assume a uniform bound T_s to coherent post-event bootstrap whenever the epoch persists that long. An intervening event cancels the pending bootstrap and starts a new waiting interval. Thus each waiting interval lasts at most T_s , or is preempted earlier. This explicitly covers rapid successive events. The correct event/cohort service, complete sampling, delivery, validation and coherent application are counted assumptions, not derived Byzantine or radio services.

During waiting, averaging and subtraction give

$$\begin{aligned}
\ddot{\xi}_{I'} + C\dot{\xi}_{I'} &= \bar{d}_{I'}, \\
\ddot{\eta}_i + C\dot{\eta}_i &= d_i - \bar{d}_{I'}, \\
\ddot{e}_j + C\dot{e}_j &= d_j - \bar{d}_{I'}.
\end{aligned} \tag{17}$$

No shielded absolute residual enters this mean law. Zero correction changes estimator/controller state but never resets the plant. It trades a finite capture pause for safe epoch semantics; it does not erase causal influence already stored in the physical or priority state.

IV. FINITE-EVENT CAPTURE AND RELEASE

A promotion gate uses thresholds $\varepsilon, \nu > 0$. In one unchanged shield epoch, it requires every proposed $j \in P$ to satisfy $\|e_j(s)\| \leq \varepsilon$, $\|\dot{e}_j(s)\| \leq \nu$ throughout $[T - H, T]$, after coherent bootstrap. The current correction and every relevant pending pre-event source, as defined in Section 3, lie in that safe epoch/window. Permanently ineligible historical tables are not inputs to the certificate and cannot block this gate. Promotion then invalidates all preceding corrections and pending eligibility as in Section 3. A gate is an existential or externally certified condition: residual origins $h = x - r - q$ alone do not provide a coherent current physical/velocity history. No online detector is implemented or inherited. A causal detector needs its own observation, communication, error and certificate contract.

Theorem 2. Conditional finite-event continuation

Suppose initial active incarnations and arrivals are finite, each incarnation follows the one-promotion/no-demotion policy, departures are terminal, every event leaves $I \neq \emptyset$, and promotions satisfy the gate. Assume the coherent causal service, finite actual initial states, ordinary forcing/goal regularity, and the gain contract. After the last exogenous event and any actual optional promotions, assume final-support priority and differential-forcing premises of Theorem 1. Then every finite prefix of the supplied admissible execution has finite nominal plant state, surviving position/velocity is continuous, Lemmas 1–2 and the waiting identities hold in their respective phases, and a continued final shield epoch captures its remaining newcomers. If any remain, a finite final group release can be selected. With no later events and the corresponding final ordinary service, its actual final relative geometry recovers without a plant reset.

Proof:

The number of born incarnations is finite. Each can promote once and depart once, so promotions and departures add only finitely many events; they are not excluded merely because exogenous events ended. Between prescribed locally finite sampling/commit/priority events, the nominal plant with held corrections is linear with locally integrable forcing. Event maps are finite and have nonzero protected denominators; induction establishes finite-prefix continuation for the supplied causal execution. This statement does not establish well-posedness of an unspecified state-dependent event or sensing rule.

After the actual last optional event, either no newcomer remains and the law becomes ordinary after bounded bootstrap,

or a fixed shield epoch remains. Choose its actual complete-history/priority cut and apply Theorem 1 to the continued shield law. Convergence and the finite newcomer set supply a finite S after which all $e_j, \dot{e}_j$ satisfy the thresholds at every time. Choose a finite $T > S + H$, after coherent bootstrap and any required preannouncement. The whole $[T - H, T]$ is safe. The current used source has age below H ; every relevant pending source does too by Section 3's explicit eligibility rule. All these sources therefore lie in the safe window. Any older retained table is permanently ineligible for both use and certification; local finiteness alone is not used to establish this fact. Select one final promotion $P = J$, which invalidates all pre-event correction eligibility. This is a permitted choice, not a claim that an unspecified online policy detects it.

After this last release, set $I^+ = A$, $J^+ = \emptyset$, invalidate pre-event corrections, wait at most T_s , and resume the ordinary full-A centered law. After another actual complete-history cut its relative equations are the parent form. Since all pre-release $d_k - \bar{d}_I \rightarrow 0$, averaging those differences gives $d_k - \bar{d}_A \rightarrow 0$. Role promotion does not change the final priority support, so shared goals still agree. The parent/Theorem 1 argument on the final ordinary coordinates proves their recovery from the actual retained history. ■

For prescribed causal source/priority/event inputs the nominal continuation is determined by its linear flow and ordered updates. Arbitrary coupled state-dependent service rules still require their own existence/uniqueness contract. Finiteness of population changes alone supplies neither priority connectivity nor physical recovery.

Partial promotion is different from final ordinary release. With $P \subsetneq J$, set $I' = I \cup P$, $J' = J \setminus P$, start the new waiting epoch, then center every active correction on I' . The remaining shielded bodies continue their tracking law. Applying ordinary full-A centering instead would let their residuals enter the new protected mean.

V. UNIFORM PROTECTED-RELATIVE AND EVENT-SEAM BOUNDS

“No direct residual term” and “uniformly independent of original distance” are distinct. For a uniform claim fix a family with: a finite uniform bound M on all birth/departure/promotion events; a finite uniform active population bound; protected initial relative positions/velocities and actual initial history bounds independent of newcomer distances; common fixed a, b, H, T_s with $\kappa < 1$; a shared uniform anchor-hull bound Q ; uniform differential disturbance bound D in every relevant epoch; fixed gate thresholds; and valid coherent source/priority contracts. Newcomer initial positions/velocities are arbitrary finite values and may vary. Timing and priority schedules may vary, including capture times growing with distance, provided these uniform conditions remain true. The compared family does not gain a hidden distance-dependent service or forcing budget.

Proposition 1. Uniform finite-event relative envelopes

Under this family, protected relative positions/velocities in shield and waiting phases, and the protected event-coordinate

seams, have finite bounds depending on these declared data and M , independent of the shielded initial distances. The bound is not uniform as $M \rightarrow \infty$.

Proof:

Induct over epochs. At a protected-set event, Lemma 2 bounds the shift using the preceding protected envelope, gate errors and the nonzero $|I'|$. Every surviving or promoted new protected coordinate is therefore uniformly bounded; no unpromoted far coordinate enters this step.

Fix a reference time scale $\tau_0 > 0$ and $Y = \|y\| + \tau_0 \|\dot{y}\|$ for a protected relative coordinate. During the first $W = T_s + H$ of an epoch, every used source is post-event, or correction is zero. With $F = \tau_0(KQ + D)$ and $\Lambda = 1/\tau_0 + C + \tau_0 K$, the integral equations bound the running supremum by Gronwall:

$$\sup_{0 \leq u \leq w} Y(u) \leq (Y(0) + Fw)e^{\Lambda w}, \quad 0 \leq w \leq W. \quad (18)$$

The same bound applies to an earlier preempted epoch. It uses post-event sources only and invents no newcomer pre-birth history. If the epoch continues past W , its complete actual length- H history is uniform. Theorem 1's parent small-gain bound, with $\|f\|_\infty \leq KQ + D$, then supplies a uniform envelope for arbitrarily long remaining shield flow. Homogeneous-response suprema depend continuously and linearly on the bounded cut data. A subsequent event applies the first step again. At most M iterations produce finite constants. This proves the claim, including waiting and short epochs. ■

The same argument applies after final full release: Lemma 2 bounds all newly protected cut data through the gate, and full-A differential forcing has a bound at most twice a bound relative to old I. A uniform finite common *physical excursion* additionally needs control of mean forcing and accumulated mean/coordinate responses. It is not implied by the relative envelope. A common force can generate unbounded translation while every relative error is zero.

For M protected events, define their actual nonnegative bounds c_r^x, c_r^v by Lemma 2's right sides. Then $\sum_r \|\Delta \xi_r\| \leq \sum_r c_r^x$ and likewise for velocity. These are coordinate-seam totals; they are not a sum of each body's physical displacement.

VI. FINAL GROUP-RELEASE MEAN RESPONSE

At a final release $P = J$ with $n = |I|$, $p = |P|$, no simultaneous departure and no later cohort/role event, let $\xi_+ = \bar{p}_{I \cup P}$. Continue the old protected analytical free mode from its actual $\xi_I(T), \bar{v}_I(T)$ under future $\bar{d}_I$, calling it ξ_I^{ref} . It is a comparison trajectory, not an injected control input. Both the post-event zero-correction wait and the subsequent coherent ordinary law have zero mean restoring force over $I \cup P$.

Theorem 3. Exact mean seam and envelope

Put $y = \xi_+ - \xi_I^{\text{ref}}$, $E = \sum_{j \in P} e_j(T)$, $V = \sum_{j \in P} \dot{e}_j(T)$, and $F_P(s) = \sum_{j \in P} (d_j(s) - \bar{d}_I(s))$. Then

$$y(t) = \frac{E}{n+p} + \frac{1 - e^{-C(t-T)}}{C(n+p)} V + \frac{1}{C(n+p)} \int_T^t (1 - e^{-C(t-s)}) F_P(s) ds. \quad (19)$$

If $F_P \in L^1([T, \infty))$, then

$$\sup_{t \geq T} \|y(t)\| \leq B_{\text{seam}}, \quad B_{\text{seam}} := \frac{\|E\| + \|V\|/C + \|F_P\|_{L^1}/C}{n+p} \leq \frac{p\varepsilon + p\nu/C + \|F_P\|_{L^1}/C}{n+p}. \quad (20)$$

and $y_\infty = (E + V/C + \int_T^\infty F_P/C)/(n+p)$. This is a finite difference limit even if the two means separately drift. Finite physical endpoints additionally require the parent mean-forcing condition.

Proof:

Averaging both laws gives $\ddot{y} + C\dot{y} = F_P/(n+p)$, with initial data $E/(n+p), V/(n+p)$ from Lemma 2. Solve the damped velocity equation and integrate. The position-force kernel is bounded by $1/C$; dominated convergence under L^1 proves the limit and the triangle inequality proves the envelope. ■

For an actual original protected body after release, $x_i = r_i + \xi_+ + \eta_i^+$. Its deviation from the continued old free formation includes both y and η_i^+ ; a mean seam alone does not bound that body's whole path. Final relative recovery removes the latter term only in the limit. With zero differential mean force, exact zero limiting seam needs $E + V/C = 0$; waiting until arbitrary positive thresholds hold does not ensure that equality.

Proposition 2. Limiting shifts versus whole responses

For a declared sequence of such mean responses with release times T_r , define each y_r on the common domain $\mathbb{R}$ by setting it to zero for $t < T_r$ and using its comparison response for $t \geq T_r$. This extension is bookkeeping, not a pre-release plant trajectory. Bounds $\|y_{r,\infty}\| \leq s_r$ with $\sum_r s_r < \infty$ bound the defined sum of *limiting shifts*. They do not bound the transient response sum. Bounds $\sup_t \|y_r(t)\| \leq b_r$, over this common domain, with $\sum_r b_r < \infty$ instead imply a uniformly convergent response series whose norm is at most $\sum_r b_r$. Theorem 3 supplies an eligible b_r when each response's hypotheses hold.

Proof:

Absolute summability and triangle inequality prove the first assertion for the limits and the third for whole responses, with the uniform Cauchy bound $\sup_t \|\sum_{r>N} y_r(t)\| \leq \sum_{r>N} b_r$. For the distinction take zero forcing, $y(T) = a \neq 0$, $\dot{y}(T) = -Ca$. The exact solution is $ae^{-C(t-T)}$: its limiting shift is zero while its trajectory is nonzero. Thus a whole-response bound by the sum of limiting bounds fails already for one response. ■

This proposition concerns a defined sum of comparison responses. It is not a theorem that an arbitrary infinitely changing

cohort decomposes into those responses, nor an infinite-event capture guarantee. A bound $s_r \leq s > 0$ alone gives no convergence of a limiting-shift series; constant positive scalar shifts are a counterexample to that inference, not a claim that every admission execution realizes them.

VII. VERIFICATION AND SCIENTIFIC POSITION

The initial 0.1 manuscript reports 10,000 centering, 10,000 transition and 5,000 telescoping checks without its script, seed or result package. Those remain unverified inherited reports; they are not authenticated by reproducing the same counts. This revision supplies **new** COSET exact rational checks with seed 20261007: 10,000 centering/cancellation cases including changed shielded positions, 10,000 protected-set position/velocity transitions, and 5,000 fixed-position telescoping sequences. Separate controls detect stale centering, partial-promotion misuse, zero-limit/nonzero-transient seams and structural epoch eligibility; valid fresh-centering and matching-epoch contrasts pass. The code and machine-readable results identify their exact revisions. These finite algebraic tests are not delayed-flow simulation, complete transport/gate implementation, physical observation or general proof. TORSION separately reports 4,000 centering, 4,000 transitions and 2,000 seam checks at its frozen reviewer frame; those remain attributed peer evidence with declared shared sources.

The previous simulation test matrix was a proposal, not a completed suite. Second arrival, protected/shielded departure, combined departure/promotion, stale frames, distance sweeps and finite mixed histories are relevant future nominal implementation checks. No existing simulator is changed or certified by this article. A later implementation must map stored epochs, partitions, source ages, pending commits, actual physical state and priority prefixes to the stated service. Its observation certificate and state-dependent event rules need their own bridge.

The arithmetic mean projection and stable convolution machinery are standard, not novelty claims. The closest internal connection is [1]’s shield algebra/capture/release argument; [2]’s service and delayed-recovery mechanism and [3]’s causal priority history are actual dependencies. [4] supplies support-map semantics, not delayed convergence. The sampled latch [5] is a different common-mode system and is excluded from composition; [6]’s conditional-query/memory results are context only. The fixed-epoch averaging note [7] provides the comparator’s admissibility, threshold and scope mechanisms rather than a shield proof.

External work already studies delay-robust consensus under broader regimes [8] and open double-integrator formation reconfiguration with barrier-based safety, a formation manager and switching conditions [9]. The relevant model/reconfiguration sections were inspected for bounded comparison. Those results are not imported to establish this article’s safety or service; this article’s particular protected-centering and coherent waiting construction retains different assumptions. No first-publication, general superiority or exhaustive novelty conclusion is asserted.

VIII. PRESERVATION, LIMITS AND NEXT BOUNDARY

The old shield embeds algebraically when $|J| = 1$ and I is fixed: its full-centered newcomer correction is $(n+1)m_j/n = h_j - \bar{h}_I$, and its protected correction is $m_i - \bar{m}_I$. With the same coherent post-bootstrap inputs and initial history, the laws coincide, so that phase’s cancellation and conditional relative recovery are preserved. The new zero-correction epoch wait differs from [1]’s retained incumbent correction prefix, and the final release reacquires its table. Whole pre-bootstrap/post-switch trajectories are therefore not declared identical to the predecessor. This is a conditional extension of the shield guarantees with an explicit service/waiting trade-off, not an unqualified stronger realization of every predecessor branch; the optional impulse and exact-rigidity branches are not imported.

Proved: modal cancellation, exact cohort-coordinate maps, conditional fixed/final recovery, uniform finite-event protected-relative envelopes under the declared family, and the final mean-response formulas and summability distinction.

Assumed: exact nominal equal-gain plant; fixed offset/anchor identities; correct coherent service, event ordering and source eligibility, including permanent pending-age expiry and older-source supersession for use and certification; bounded age/bootstrap, actual priority packet/weight/connectivity contracts and physical embedding; factored-gain small-age stability; finite lifecycle events; stated disturbance/history and uniform-family bounds; an externally satisfied release gate.

Surviving guarantees and trade-offs: no plant reset, no direct unpromoted residual in protected mean feedback, controlled gated coordinate seams, and final relative recovery survive. Waiting, complete origins and finite-lifecycle book-keeping pay for changed cohorts. No unchanged full-trajectory comparison, finite common endpoint without mean integrability, or whole-motion bound from limiting seams survives as an unconditional claim.

Next attack: a causal observation/certificate interface for release is the nearest missing implementation bridge. Composition with the sampled latch, infinite-event tracking/summable budgets, finite precision and physical approach feasibility remain distinct deferred problems. No new theorem or experiment for those problems is started by naming them.

No collision avoidance, actuator/speed/energy bound for arbitrary far newcomers, distance-independent capture deadline, arbitrary radio delivery, Byzantine membership, rigid-body flight, historical priority restoration, gain/offset edits or perpetual-churn convergence is established. Exact geometry and nominal feasibility do not establish real-world safety. Final acceptance is scoped to the two permanent AI reviewers’ exact artifact frame; human final acceptance and any dissemination remain separate.

IX. AI-USE DISCLOSURE

This work was developed with assistance from AI agents operating within the HORTAME Systems research architecture. Depending on the research stage, these agents contributed to literature analysis, mathematical formulation, proof

reconstruction, adversarial review, counterexample search, simulation and code analysis, verification design, reproducibility checks, revision, and manuscript preparation. AI-agent outputs were treated as research artifacts subject to verification and correction rather than as independent authority. G. Bolduc is the human author responsible for the manuscript, its claims, source selection, and publication decisions. AI-agent review does not constitute independent human peer review, proof-assistant certification, experimental validation, or publication acceptance.

COSSET studied the seven supplied originals, derived the initial assessment and fresh exact checks, and revised the manuscript. TORSION separately studied the same originals and froze its initial assessment before comparing findings, then reviewed affected corrections. Their shared model/source/protocol lineage limits independence. These statements record actual performed contributions; final-frame acceptance is recorded separately rather than inferred here.

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